00Name:		Centre/Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination 2025
Year 6

FURTHER MATHEMATICS 9649/02

Paper 2

23 September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Write your name, centre number, index number and class on this question paper.

An answer booklet will be provided with this question paper. You should follow the instructions on the front of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [50 marks]

1 The largest positive root, α, of $5x-x^3=\mathrm{e}^{-x}$ falls in the interval $N \le x \le N+1$, where N is an integer.

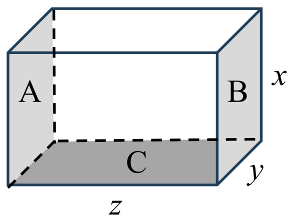
(a) By sketching the graphs of $y=5x-x^3$ and $y=\mathrm{e}^{-x}$, show that $N=2$. [3]

(b) Use linear interpolation once on the interval $[N, N+1]$, with $\mathrm{f}(x)=5x-x^3-\mathrm{e}^{-x}$ to find an approximation to α, giving your answer correct to 5 decimal places. [2]

The root α for the equation $5x-x^3=\mathrm{e}^{-x}$ satisfies the equation $x=\mathrm{g}(x)$, where $\mathrm{g}(x)=\left(5x-\mathrm{e}^{-x}\right)^{\frac{1}{3}}$. An iterative method based on the form $x_{n+1}=\mathrm{g}(x_n)$ can be used to obtain an approximation for α.

(c) Use the answer in part **(b)** as the initial approximation for α to determine the value of α, correct to 4 decimal places. [3]

2 An open box is made of cardboard with the dimensions x, y, z units as shown in the diagram. The box has a fixed volume of 3 units^3. To make the box sturdy, the sides of the box labelled A and B are added with one extra layer of cardboard while the bottom of the box labelled C is added with two extra layers of cardboard. The total area of cardboard required to construct the box is denoted by S units^2. You may assume that the thickness of the cardboard is negligible.



(a) Find the dimensions of the box such that S is minimum. [6]

(b) By differentiating S with respect to t, find the rate of change of S if x and y is increasing at a constant rate of $\frac{1}{2}$ units per second and $-\frac{1}{3}$ units per second respectively when $x=1$, $y=3$ and $z=1$. [3]

3 **(a)** Let $\mathrm{M}:\mathbb{R}^3 \to \mathbb{R}^2$ be defined by

$$\mathrm{M}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b \\ c \end{pmatrix}.$$

Show that M is a linear transformation. [2]

(b) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Let $T^m = \underbrace{T \circ T \circ \cdots \circ T}_{m \text{ times}}$ denote the composition of the linear transformation T with itself m times. It is known that

$$N_{k-1} \subseteq N_k \text{ --- } (*)$$

for all positive integers k, where N_m is the null space of T^m.

(i) Use (*) to prove that if the null space of T^{k-1} is equal to the null space of T^k for some positive integer k, then the null space of T^k is equal to the null space of T^{k+1}. [3]

(ii) Given that m is the smallest positive integer such that $T^m(\mathbf{w}) = \mathbf{0}$ for all $\mathbf{w} \in \mathbb{R}^3$ and the nullity of T is non-zero, explain why $m \le 3$. [2]

4 An oncologist is studying the growth of a tumour in one of her patients. The oncologist believes the size of the tumour, P (in millimetres), at the time t months after it was first diagnosed can be modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\left(1 - \frac{P}{N}\right),$$

where k and N are constants.

(a) State, in context, the significance of the constants k and N in the model. [2]

(b) Given that P_0 is the initial size of tumour when it was first diagnosed, show that an expression for the size of the tumour P after t months is $\dfrac{P_0 N}{P_0 + (N - P_0)\mathrm{e}^{-kt}}$. [4]

(c) The oncologist needs to analyse the tumour's growth pattern to determine the best course of action for treatment. She found that the tumour size has grown to 5 times its original size at the end of 2 months. Assuming that $N = 25P_0$, find the exact value of k. [2]

After analysing the tumour growth, the oncologist decides to start her patient on chemotherapy. The growth rate of the size of tumour now follows the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\left(1 - \frac{P}{N}\right) - HP,$$

where H, the therapy induced death rate, is a positive constant.

(d) Given that the tumour was detected early, find the range of values of H, in terms of k, for the tumour to shrink in size and eventually be eradicated. [4]

5 A colony of bacteria is growing on an agar plate. It is estimated that the initial population of bacteria on the plate was 6000 and that the population of bacteria at the end of the first day was 9000.

From previous studies, it is known that the increase in bacteria population from day n to day $(n+1)$ is twice the increase from day $(n-1)$ to day n.

(a) **(i)** Let b_n denote the population of bacteria at the end of the nth day. Find an expression for b_n in terms of n. [5]

(ii) Deduce the predicted size of the bacteria population at the end of the 6th day. [1]

Another agar plate containing a colony of the same bacteria with same initial population was contaminated with a virus which kills the bacteria while it was trying to grow. The population of bacteria c_n for this plate can be modeled by the recurrence relation

$$c_n = \frac{3c_{n-1} - 2c_{n-2}}{1+k}$$

where $0.2 < k < 1$ and c_n denotes the population of bacteria at the end of nth day.

(b) Given that the population of the bacteria at the end of the first day was 9000, and that the population of the bacteria was eliminated at the end of the 6^{th} day, find the value of k. [8]

Section B: Probability and Statistics [50 marks]

6 A particular machine part in a factory is known to fail over time more rapidly due to aging. Its reliability is measured in the number of hours of use before it needs to be replaced. The manufacturer of the machine part models its reliability by the random variable R (in hours) with cumulative distribution function.

$$\mathrm{F}(r) = \begin{cases} 1 - \mathrm{e}^{-\left(\frac{r}{3200}\right)^{3.2}} & , \quad r \geq 0 \\ 0 & , \quad r < 0 \end{cases}$$

(a) Find the probability that the machine part will last more than another 1400 hours given that it has lasted more than 2000 hours. [2]

The factory owner who purchases large quantities of the machine part from the manufacturer recorded the reliability of a random sample of 210 of these parts over a period of time with the observed frequencies as follows

Reliability, R (hours)	**Observed Frequency**
$R < 2000$	53
$2000 \le R < 2500$	41
$2500 \le R < 3000$	β
$3000 \le R < 3500$	26
$3500 \le R < 4000$	$70 - \beta$
$4000 \le R < 4500$	16
$4500 \le R < 5000$	3
$R \ge 5000$	1

(b) Given that a chi-square goodness of fit test at the 5% significance level concluded that the data is not consistent with the manufacturer's model, find the range of values of β. [5]

7 A language learning app is testing whether its new AI pronunciation coach improves users' pronunciation fluency. Twelve users were scored on a scale of 0 to 100 (with each score given to 1 decimal place) before and after using the app for 2 weeks. A higher score indicates better pronunciation fluency.

User	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Before	70.0	65.5	75.1	68.0	72.2	69.8	66.4	75.0	67.0	73.0	70.1	64.5
After	73.2	69.0	72.4	64.3	76.0	72.7	71.9	72.0	66.1	77.6	67.8	70.7

(a) Explain why a test based on the t-distribution might not be suitable in this context. [1]

(b) Carry out a suitable Wilcoxon test, using a 5% level of significance, to show that there is insufficient evidence to claim that the app is effective in improving users' pronunciation fluency. [4]

(c) It was subsequently discovered that User 11's score after the use of the app was recorded wrongly. With the correct score in place, the Wilcoxon test will now lead to the conclusion that there is sufficient evidence at 5% level of significance that the app is effective in improving users' pronunciation fluency. Given that the rank for each user remains unchanged, determine the lowest possible correct score for User 11 after using the app. Explain how you arrive at your answer. [2]

8 Mrs Yeo occasionally allows her son to drive her car. After Mrs Yeo drives the car, the amount of petrol in the tank is uniformly distributed between 10 litres and 50 litres. After her son drives the car, the amount of petrol in the tank is uniformly distributed between 0 litres and 20 litres. Mrs Yeo is the driver for 80% of the time and her son is the driver for the remaining 20% of the time.

(a) Mrs Yeo checks the car at home, not knowing who drove it last. Find the probability that there is less than 15 litres of petrol in the tank. [2]

Let T be the random variable denoting the amount of petrol in the tank in litres after someone used the car.

(b) Find the cumulative distribution function of T. Hence or otherwise, find $\mathrm{E}(T)$. [4]

(c) The maximum distance, in km, of the car journey, D, depends on T such that $D = 650 - \dfrac{3250}{T+5}$. Find the probability that next driver can drive at least 504 km before it runs out of petrol. [3]

9 **(a)** Let X be a random variable that follows the Poisson distribution with mean λ.

It is given that $\sum_{r=1}^{\infty} r^2 \mathrm{P}\left(X = r\right) = 2$. Find the value of λ. [3]

(b) In a particular boarding school with a large number of students, the students do not stay with their parents. The number of telephone calls home D made by a female student each week has a Poisson distribution with mean 2, whilst the number of telephone calls home S made by a male student each week has a Poisson distribution with mean 1. A male and a female student are chosen randomly.

(i) Find the probability that both students make an equal number of calls home not exceeding 2 calls each in a week. [2]

(ii) The probability that either of them makes at least a call home within n days is more than 0.8. Find the least value of n. [3]

(iii) Find the probability that the female student makes less than 8 calls home but more than the number of calls the male student makes in a randomly chosen week. [2]

(iv) Given that, in a particular week, a total of 7 calls were made, find the probability that difference between the number of calls home made by the female and the male student is more than 3. [3]

10 Japple engages a mobile phone battery manufacturer to make the batteries for its new range of j-phones. One of the requirements is that a new battery should have a mean battery life of 10 hours under regular usage before needing to be recharged.

To find out if the manufacturer has met this requirement, Japple took a random sample of 10 new batteries and recorded the battery life (in hours) under regular usage in their j-phones as follows.

#1	8.47	**#2**	11.36	**#3**	10.26	**#4**	9.59	**#5**	7.38
#6	7.38	**#7**	6.89	**#8**	10.93	**#9**	9.61	**#10**	10.14

(a) Stating any assumption(s) required, find a 95% confidence interval for the mean battery life, leaving your answers to 3 decimal places. Working should be shown. [3]

Japple decides to take another random sample of 8 new batteries and repeat the test. For this second sample, a 95% confidence interval works out to be (9.015, 9.985) (in hours).

(b) With the same assumption as in **(a)**, calculate a 95% confidence interval for all 18 batteries in the 2 samples, leaving your answers to 3 decimal places. [6]

(c) Deduce the conclusion of a hypothesis test for the combined sample, at 5% level of significance, on whether the mean battery life is 10 hours. [1]

Another requirement of the new batteries is that they must have no difference in their battery life within 50 cycles of full charging. Japple takes the first sample of 10 batteries, subjected them to 50 cycles of full charging and then recorded their battery life (in hours) after the 50^{th} charge as follows:

#1	8.70	**#2**	11.30	**#3**	10.22	**#4**	9.61	**#5**	7.70
#6	8.10	**#7**	7.86	**#8**	10.70	**#9**	9.90	**#10**	9.98

(d) Assuming that the difference in battery lives before and after the 50 cycles of full charging of a j-phone is normally distributed, carry out a suitable test at 3% level of significance and state your conclusion clearly. [4]

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