Electric and Magnetic Fields tutorial

1 The six faces of a cubical box each measure 20.0 cm by 20.0 cm, and the faces are numbered such that faces 1 and 6 are opposite to each other, as are faces 2 and 5, and faces 3 and 4. The flux through each face is given in the table. Determine the net charge inside the cube.

face	flux / N m^2 C^{-1}
1	–70.0
2	–300.0
3	–300.0
4	300.0
5	–400.0
6	–500.0

2 Electric fields of varying magnitudes are directed either inward or outward at right angles on the faces of a cube, as shown in Fig. 2.1.

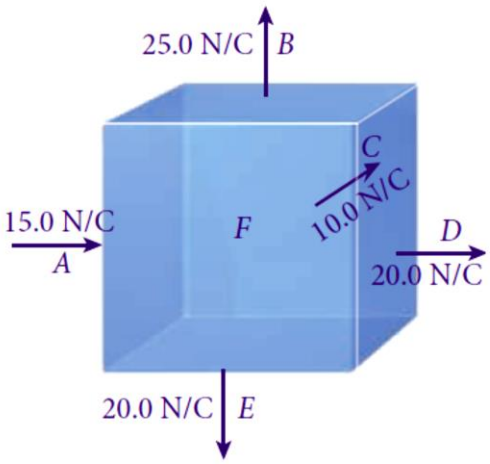


Fig. 2.1

Determine the strength and direction of the field on the face *F*.

3 A –6.00 nC point charge is located at the centre of a conducting spherical shell. The shell has an inner radius of 2.00 m, an outer radius of 4.00 m, and a charge of 7.00 nC.

(a) Determine

(i) the electric field at $r = 1.00$ m,

(ii) the electric field at $r = 3.00$ m,

(iii) the electric field at $r = 5.00$ m.

(b) Calculate the surface charge distribution, σ, on the outside surface of the shell.

4 A hollow conducting spherical shell has an inner radius of 8.00 cm and an outer radius of 10.0 cm. The electric field at the inner surface of the shell has a magnitude of 80.0 N C^{-1} and points toward the centre of the sphere, and the electric field at the outer surface has a magnitude of 80.0 N C^{-1} and points away from the centre of the sphere. Determine the charge on the inner surface and on the outer surface of the spherical shell.

5 Two parallel, infinite, non-conducting plates are 10.0 cm apart and have charge distributions of 1.00 μC m^{-2} and –1.00 μC m^{-2}. Calculate the force on an electron

(a) in the space between the plates,

(b) located outside the two plates near the surface of one of the two plates.

6 Two parallel, uniformly charged, infinitely long wires are 6.00 cm apart and carry opposite charges with a linear charge density of 1.00 μC m^{-1}. Determine the magnitude and direction of the electric field at a point midway between the two wires and 40.0 cm above the plane containing them.

7 A thin, hollow, metal cylinder of radius R has a surface charge distribution σ. A long, thin wire with a linear charge density $\frac{\lambda}{2}$ runs through the centre of the cylinder. Determine an expression for the electric field and determine the direction of the field at each of the following locations:

(a) $r < R$

(b) $r \geq R$

8 Two infinite sheets of charge are separated by 10.0 cm as shown in Fig. 9.1. Sheet 1 has a surface charge distribution of $\sigma_1 = 3.00\ \mu\text{C m}^{-2}$ and sheet 2 has a surface charge distribution of $\sigma_1 = -5.00\ \mu\text{C m}^{-2}$. Determine the total electric field at

(a) P, 6.00 cm to the left of sheet 1,

(b) P', 6.00 cm to the right of sheet 1.

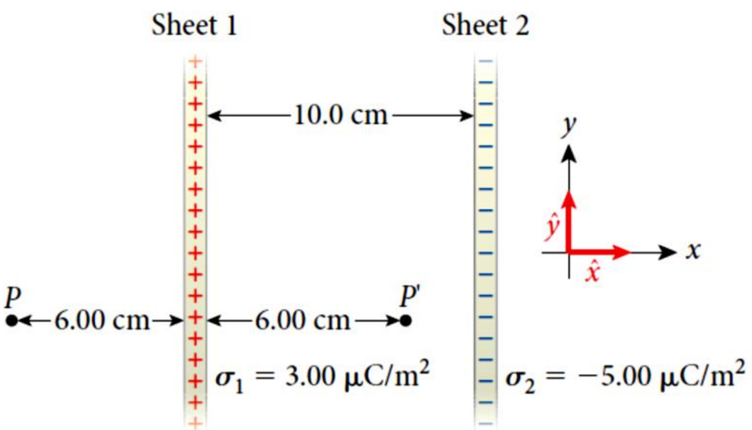


Fig. 9.1

9 A non-conducting sphere has a uniform volume charge density ρ. Let $\vec{r}$ be the vector from the centre of the sphere to a general point P within the sphere.

(a) Show that the electric field at P is given by $\frac{\rho\vec{r}}{3\varepsilon_0}$. (Note that the result is independent of the radius of the sphere.)

(b) A spherical cavity is hollowed out of the sphere, as shown in Fig. 11.1.

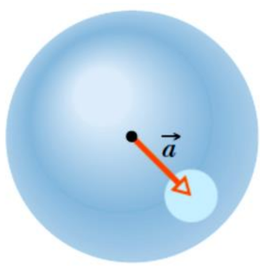


Fig. 11.1

Using superposition concepts, show that the electric field at all points within the cavity is uniform and equal to $\dfrac{\rho\vec{a}}{3\varepsilon_0}$, where $\vec{a}$ is the position vector from the centre of the sphere to the centre of the cavity. (Note that this result is independent of the radius of the sphere and the radius of the cavity)

10 Suppose the charge density of a solid sphere in is given by $\rho = \alpha r^2$ where α is a constant.

(a) Determine α in terms of the total charge Q on the sphere and its radius r_0.

(b) Determine the electric field as a function of r inside the sphere.

11 State a similarity and two differences between the application of Gauss's and Ampere's laws.

12 The current I within a wire that has a circular cross section of radius R is known to be distributed uniformly over that cross section.

(a) Determine the magnetic field as a function of the distance r from the wire's axis where

(i) $r < R$

(ii) $r > R$

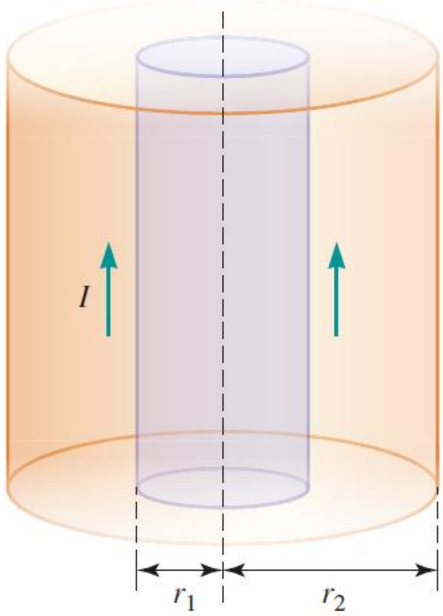
(b) Sketch the $B - r$ graph.

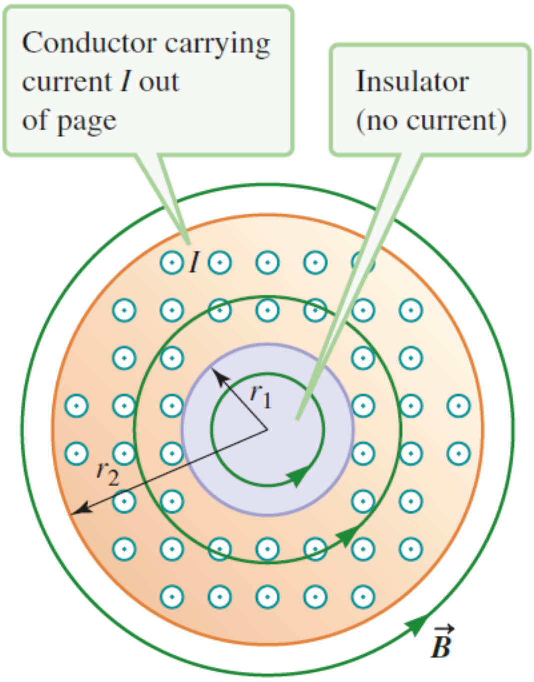
13 A very long, cylindrical wire has an insulating core surrounded by a cylindrical conductor carrying current I. The current is uniformly distributed in the conducting cylinder. The radius of the insulating core is r_1, and the outer radius of the conducting cylinder is r_2. Determine expressions for the magnetic flux density

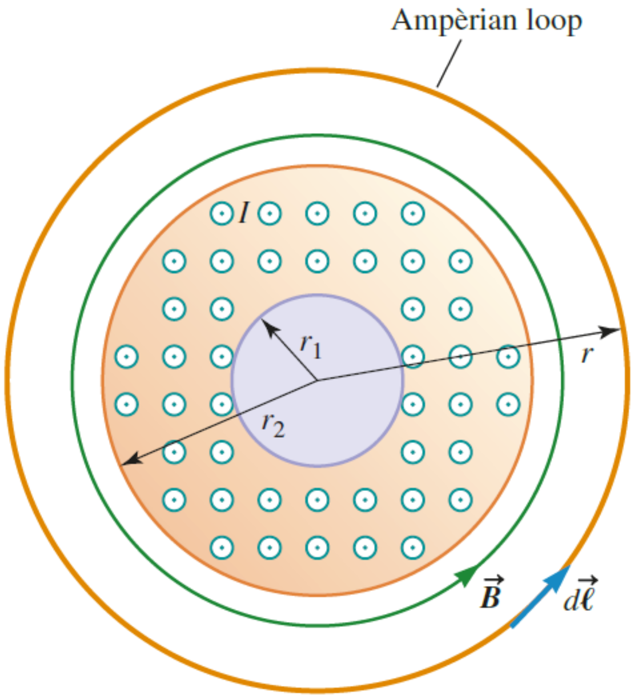
(a) outside the conducting cylinder ($r > r_2$),

(b) inside the conducting cylinder but outside the insulator ($r_1 < r < r_2$),

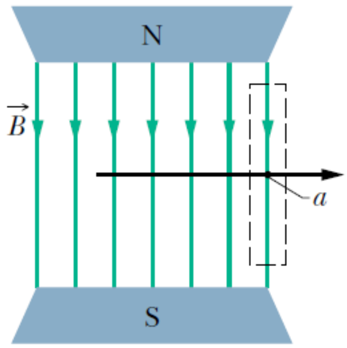
(c) and inside the insulator ($r < r_1$).







14 Show that a uniform magnetic field cannot drop abruptly to zero.



15 A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of magnitude 0.800 A. Determine the magnitude of the magnetic field inside the toroid at

(a) the inner radius and

(b) the outer radius of the toroid.

16 The current density in a cylindrical conductor of radius R varies as $J(r) = \frac{J_0 r}{R}$ (in the region from zero to R).

(a) Express the magnitude of the magnetic field, B, in the regions $r < R$ and $r > R$.

(b) Sketch the graph of B against r.

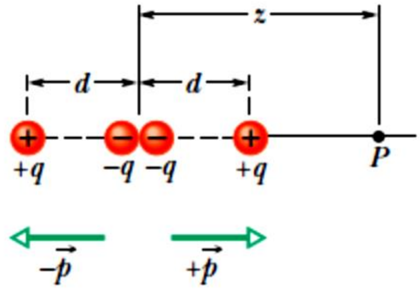
17 A current loop, carrying an amount of current of 5.0 A, is in the shape of a right triangle with sides 30 cm, 40 cm, and 50 cm. The loop is in a uniform magnetic field of magnitude 80 mT whose direction is parallel to the current in the 50 cm side of the loop. Calculate the magnitude of the

(a) magnetic dipole moment of the loop,

(b) torque on the loop.

18 Calculate the magnitude of the force, due to an electric dipole of dipole moment 3.6×10^{-29} C m, on an electron 25 nm from the centre of the dipole, along the dipole axis. Assume that this distance is large relative to the dipole's charge separation.

19 Figure below shows an electric quadrupole. It consists of two dipoles with dipole moments that are equal in magnitude but opposite in direction. Determine the magnitude of $\vec{E}$ on the axis of the quadrupole at a point *P* a distance *z* from its centre. Assume $z \gg d$.



20 An electric dipole consists of charges 2*e* and –2*e* separated by 0.78 nm. It is in an electric field of strength 3.4×10^{6} N C^{-1}. Calculate the magnitude of the torque on the dipole when the dipole moment is

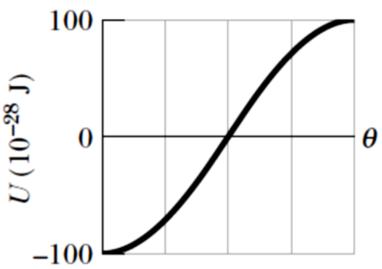
(a) parallel to,
(b) perpendicular to, and
(c) antiparallel to the electric field.

21 Sketch the graph of

(a) τ against θ
(b) U against θ

of an electric dipole placed in a uniform electric field, where τ is the torque, U is the electric potential energy and θ is the angle between the dipole moment and the field.

22 Figure below shows the potential energy U of an electric dipole against the angle θ between a uniform electric field $\vec{E}$ of magnitude 20 N C^{-1} and the dipole moment $\vec{p}$.



Calculate is the magnitude of $\vec{p}$.

23 Determine the frequency of oscillation of an electric dipole, of dipole moment $\vec{p}$ and moment of inertia I, for small amplitudes of oscillation about its equilibrium position in a uniform electric field $\vec{E}$.

24 An electric dipole consists of an electron and a proton at a separation of 2.00 nm and it is in a uniform field of magnitude 3.00×10^{6} N C^{-1}. Calculate the amount of energy required to turn an electric dipole from being lined up with the electric field to being lined up opposite the field.

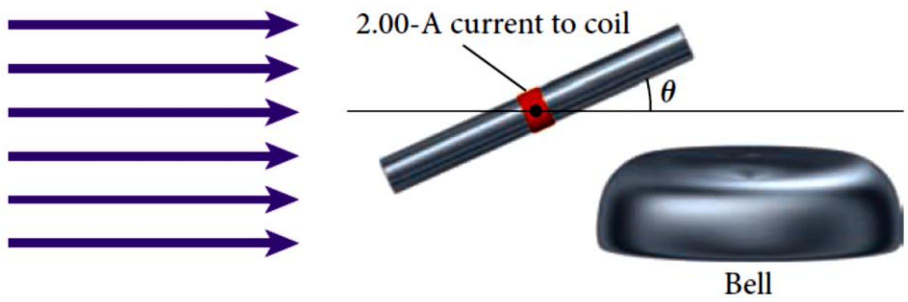
25 An infinite line of positive charge lies along the y axis, with charge density $\lambda = 2.00 \mu\text{C m}^{-1}$. A dipole is placed with its center along the x axis at $x = 25.0$ cm. The dipole consists of two charges of magnitude 10.0 μC separated by 2.00 cm. The axis of the dipole makes an angle of 35.0° with the x axis, and the positive charge is further from the line of charge than the negative charge. Calculate the net force exerted on the dipole.

26 A circular wire of radius 5.0 cm has a current of 3.0 A flowing in it. The wire is placed in a uniform magnetic field of 5.0 mT.

(a) Determine the maximum torque on the wire.

(b) Determine the range of the magnetic potential energy of the wire.

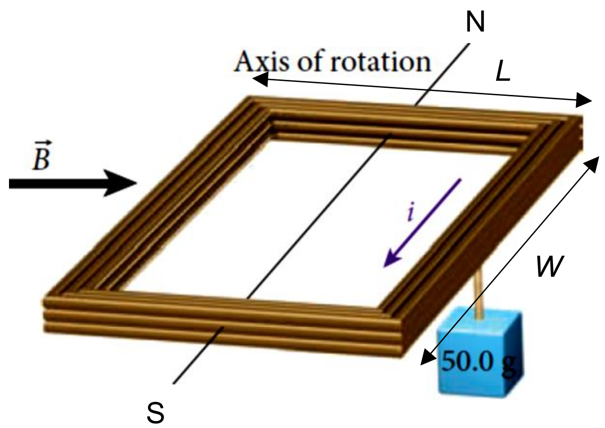
27 An electromagnetic doorbell has 70 turns of wire around a long, thin rod, as shown. The rod has mass 30.0 g, length 8.00 cm, and a cross-sectional area of 0.200 cm^2. The rod is free to move and is pivoted about an axis through its centre, which coincides with the centre of the coil. The rod is initially at rest when $\theta = 25.0°$ with the horizontal. When $\theta = 0.00°$, the rod strikes a bell. A uniform magnetic field of 9.00×10^{-2} T is directed at an angle $\theta = 0.00°$. Given that a current of 2.00 A is in the coil, calculate



(a) the torque on the rod when $\theta = 25.0°$

(b) the angular velocity of the rod when it strikes the bell.

28 A 50-turn rectangular coil of wire with dimensions $L = 10.0$ cm by $W = 20.0$ cm lies in a horizontal plane, as shown in the figure. The axis of rotation of the coil is aligned north and south. It carries a current $i = 1.00$ A and is in a magnetic field pointing from west to east. A mass of 50.0 g hangs from one side of the coil. Determine the strength the magnetic field in order to have to keep the coil in the horizontal orientation.



29 In a highly simplified model of the atom, the hydrogen atom is assumed to consist of an electron moving with speed v in a circular orbit with radius r around a stationary proton. The centripetal force keeping the electron moving in a circle is the electric force between the proton and the electron. The radius of the orbit of the electron is $r = 5.29 \times 10^{-11}$ m.

(a) Derive an expression relating the magnetic dipole moment, $\vec{\mu}$, and the angular momentum $\vec{L}$ in terms of the mass of an electron, *m*, and the elementary charge, *e*.

(b) Calculate the magnitude of the orbital magnetic moment of the hydrogen atom.

30 A magnetic dipole is in a uniform magnetic field.

(a) **(i)** Show that the magnetic dipole can oscillate as a torsional pendulum in simple harmonic motion when displaced from its equilibrium orientation and released.

(ii) State and explain an assumption made in **(i)**.

Given that the dipole is a compass needle (a light bar magnet) with a magnetic moment of magnitude μ and moment of inertia about its centre is I. It is mounted on a frictionless, vertical axle and it is placed in a horizontal magnetic field of magnitude *B.*

(b) **(i)** Determine the frequency of oscillation of the compass needle.

(ii) Explain how the compass needle can be conveniently used as an indicator of the magnitude of the external magnetic field.

(iii) If the frequency is 0.680 Hz in the Earth's local field, with a horizontal component of 39.2 μT, calculate the magnitude of a field parallel to the needle in which its frequency of oscillation is 4.90 Hz.

Answers:

1	-1.12×10^{-8} C	**16**	$\frac{\mu_0 J_0 r^3}{3R}$, $\frac{\mu_0 J R^2}{3r}$
2	-60 NC^{-1}	**17**	0.30 A m^{-2}, 0.024 N m
3	54 NC^{-1}, 0,0.36 NC^{-1}, 4.97×10^{-12} C m^{-2}	**18**	6.6×10^{-15} N
4	8.90×10^{-11} C	**19**	$\frac{3qd^2}{2\pi\varepsilon_0 z^4}$
5	1.81×10^{-14} N, 0	**20**	0, 8.5×10^{-22} N m, 0
6	6.71×10^{3} N C^{-1}, 1.50 rad		
7	$E=\frac{\lambda}{4\pi\varepsilon_0 r}$, $E=\frac{\lambda+4\pi\sigma R}{4\pi\varepsilon_0 r}$	**22**	5.0×10^{-28} C m
8	1.13×10^5 N C^{-1} to the right 4.52×10^5 N C^{-1} to the right.	**23**	$f=\frac{1}{2\pi}\sqrt{\frac{pE}{I}}$
		24	1.92×10^{-21} J
10	$\alpha=\frac{5Q}{4\pi r_0^5}$, $E=\frac{k_e Q}{r_0^5}r^3$	**25**	9.44×10^{-2} N
		26	1.2×10^{-4} N m, 2.4×10^{-4} J
12	$\frac{\mu_0 I}{2\pi R^2}r$, $\frac{\mu_0 I}{2\pi r}$	**27**	1.06×10^{-4} N m, 1.72 rad s^{-1}
13	$\frac{\mu_0 I}{2\pi r}$, $\frac{\mu_0 I\left(r^2-r_1^2\right)}{2\pi r\left(r_2^2-r_1^2\right)}$, 0	**28**	0.0245 T
		29	$\vec{\mu}=-\frac{e}{2m}\vec{L}$, 9.25×10^{-24} A m^2
15	5.33×10^{-4} T, 4.00×10^{-4} T	**30**	2.04 mT

Electric and Magnetic Fields tutorial SOLUTIONS

1 The six faces of a cubical box each measure 20.0 cm by 20.0 cm, and the faces are numbered such that faces 1 and 6 are opposite to each other, as are faces 2 and 5, and faces 3 and 4. The flux through each face is given in the table. Determine the net charge inside the cube.

face	flux / N m² C⁻¹
1	–70.0
2	–300.0
3	–300.0
4	300.0
5	–400.0
6	–500.0

Using Gauss's law,

$$\Phi_E = \frac{q}{\varepsilon_0}$$

$$\begin{aligned} q &= \Phi_E \varepsilon_0 \\ &= \left[(-70.0)+(-300.0)+(-300.0)+300.0+(-400.0)+(-500.0)\right]\left(8.85\times10^{-12}\right) \\ &= -1.12\times10^{-8}\ \text{C} \end{aligned}$$

2 Electric fields of varying magnitudes are directed either inward or outward at right angles on the faces of a cube, as shown in Fig. 2.1.

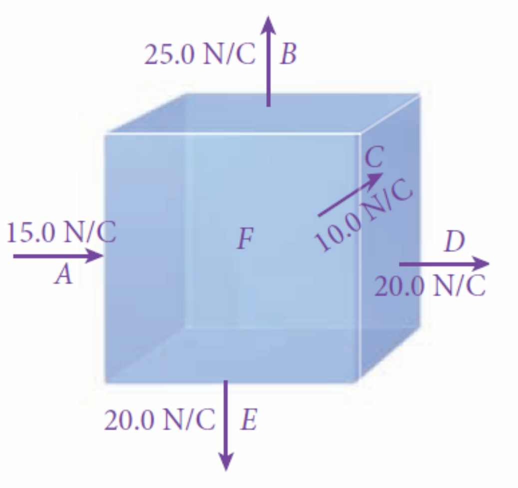


Fig. 2.1

Determine the strength and direction of the field on the face *F*.

Using Gauss's law,

$$\Phi_E = \frac{q}{\varepsilon_0}$$
$$= 0 \quad (\because\ q = 0 \text{ as the cube does not contain charge})$$
$$\vec{E}_A \cdot \vec{A}_A + \vec{E}_B \cdot \vec{A}_B + \vec{E}_C \cdot \vec{A}_C + \vec{E}_D \cdot \vec{A}_D + \vec{E}_E \cdot \vec{A}_E + \vec{E}_F \cdot \vec{A}_F = 0$$
$$(-E_A + E_B + E_C + E_D + E_E + E_F)A = 0 \quad (\text{where } A \text{ is the area of each face of the cube})$$
$$E_F = -(-E_A + E_B + E_C + E_D + E_E)$$
$$= -[(-15.0 + 25.0 + 10.0 + 20.0 + 20.0)]$$
$$= -60.0 \text{ N C}^{-1}$$

Electric field strength on face F is 60.0 N C^{-1}, at right angles inwards.

3 A –6.00 nC point charge is located at the centre of a conducting spherical shell. The shell has an inner radius of 2.00 m, an outer radius of 4.00 m, and a net charge of 7.00 nC.

(a) Determine

(i) the electric field at $r = 1.00$ m,

(ii) the electric field at $r = 3.00$ m,

(iii) the electric field at $r = 5.00$ m.

(b) Calculate the surface charge distribution, σ, on the outside surface of the shell.

(a) (i) By Gauss's law,

$$\int_A \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$
$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$
$$= \frac{6.00 \times 10^{-9}}{4\pi(8.85 \times 10^{-12})(1.00)^2}$$
$$= 54.0 \text{ N C}^{-1}$$

Electric field is 54.0 N C^{-1} radially inwards toward the centre of the spherical shell.

(ii) Electric field inside a conductor is zero.

(iii) By Gauss's law,

$$\int_A \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$
$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$
$$= \frac{[7.00 + (-6.00)] \times 10^{-9}}{4\pi(8.85 \times 10^{-12})(5.00)^2}$$
$$= 0.360 \text{ N C}^{-1}$$

(b) Charge on the outer surface, q_o,

$$q_o = (7.00 - 6.00) \times 10^{-9}$$
$$= 1.00 \times 10^{-9} \text{ C}$$

$$\sigma = \frac{q}{A}$$
$$= \frac{q_o}{4\pi r^2}$$
$$= \frac{1.00\times10^{-9}}{4\pi(4.00)^2}$$
$$= 4.97\times10^{-12}\ \text{C m}^{-2}$$

4 A hollow conducting spherical shell has an inner radius of 8.00 cm and an outer radius of 10.0 cm. The electric field at the inner surface of the shell has a magnitude of 80.0 N C^{-1} and points toward the centre of the sphere, and the electric field at the outer surface has a magnitude of 80.0 N C^{-1} and points away from the centre of the sphere. Determine the charge on the inner surface and on the outer surface of the spherical shell.

By Gauss's law,

$$\int_A \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$
$$-E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$
$$q = -4\pi\varepsilon_0 r^2 E$$
$$= -4\pi\left(8.85\times10^{-12}\right)\left(8.00\times10^{-2}\right)^2(80.0)$$
$$= -5.69\times10^{-11}\ \text{C}$$

The charge on the inner surface of the spherical shell is 5.69 × 10^{-11} C.

By Gauss's law,

$$\frac{q'}{q} = \left(\frac{r'}{r}\right)^2$$
$$q' = \left(\frac{r'}{r}\right)^2 q$$
$$= \left(\frac{10.0}{8.00}\right)^2\left(5.69\times10^{-11}\right)$$
$$= 8.90\times10^{-11}\ \text{C}$$

The charge on the outer surface of the spherical shell is 8.90 × 10^{-11} C.

5 Two parallel, infinite, non-conducting plates are 10.0 cm apart and have charge distributions of 1.00 μC m^{-2} and –1.00 μC m^{-2}. Calculate the force on an electron

(a) in the space between the plates,

(b) located outside the two plates near the surface of one of the two plates.

$$E = \frac{\sigma}{\varepsilon_0}$$
$$F = eE$$
$$= \frac{e\sigma}{\varepsilon_0}$$
$$= \frac{\left(1.60\times10^{-19}\right)\left(1.00\times10^{-6}\right)}{8.85\times10^{-12}}$$

(a) $= 1.81\times10^{-14}\ \text{N}$

(b) Electric field strength outside the two plates is zero everywhere. Hence the force is zero.

6 Two parallel, uniformly charged, infinitely long wires are 6.00 cm apart and carry opposite charges with a linear charge density of 1.00 μC m^{-1}. Determine the magnitude and direction of the electric field at a point midway between the two wires and 40.0 cm above the plane containing them.

Distance of point from wires, r,

$$r=\sqrt{\left(\frac{d}{2}\right)^2+h^2}$$

$$=\sqrt{\left(\frac{6.00\times10^{-2}}{2}\right)^2+\left(40.0\times10^{-2}\right)^2}$$

$$=40.1\text{ cm}$$

By Gauss's law,

$$\int_A \vec{E}\cdot d\vec{A}=\frac{q}{\varepsilon_0}$$

$$E(2\pi r)l=\frac{\lambda l}{\varepsilon_0}$$

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

$$=\frac{1.00\times10^{-6}}{2\pi\left(8.85\times10^{-12}\right)\left(40.1\times10^{-2}\right)}$$

$$=4.48\times10^{-4}\text{ N C}^{-1}$$

$$\tan\theta=\frac{h}{\left(\frac{d}{2}\right)}$$

$$=\frac{2h}{d}$$

$$=\frac{2(40.0)}{6.00}$$

$$\theta=1.50\text{ rad}$$

$$E_R=2E\cos\theta$$

$$=2\left(4.48\times10^{4}\right)\cos1.50$$

$$=6.71\times10^{3}\text{ N C}^{-1}$$

7 A thin, hollow, metal cylinder of radius R has a surface charge distribution σ. A long, thin wire with a linear charge density $\frac{\lambda}{2}$ runs through the centre of the cylinder. Determine an expression for the electric field and determine the direction of the field at each of the following locations:

(a) $r<R$

(b) $r\geq R$

(a) By Gauss's law,

$$\int_A \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E(2\pi r)l = \frac{\left(\frac{\lambda}{2}\right)l}{\varepsilon_0}$$

$$E = \frac{\lambda}{4\pi\varepsilon_0 r}$$

(b) By Gauss's law,

$$\int_A \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E(2\pi r)l = \frac{\left(\frac{\lambda}{2}\right)l + \sigma(2\pi R)l}{\varepsilon_0}$$

$$E = \frac{\lambda + 4\pi\sigma R}{4\pi\varepsilon_0 r}$$

8 Two infinite sheets of charge are separated by 10.0 cm as shown in Fig. 9.1. Sheet 1 has a surface charge distribution of $\sigma_1 = 3.00\ \mu\text{C m}^{-2}$ and sheet 2 has a surface charge distribution of $\sigma_1 = -5.00\ \mu\text{C m}^{-2}$. Determine the total electric field at

(a) P, 6.00 cm to the left of sheet 1,
(b) P', 6.00 cm to the right of sheet 1.

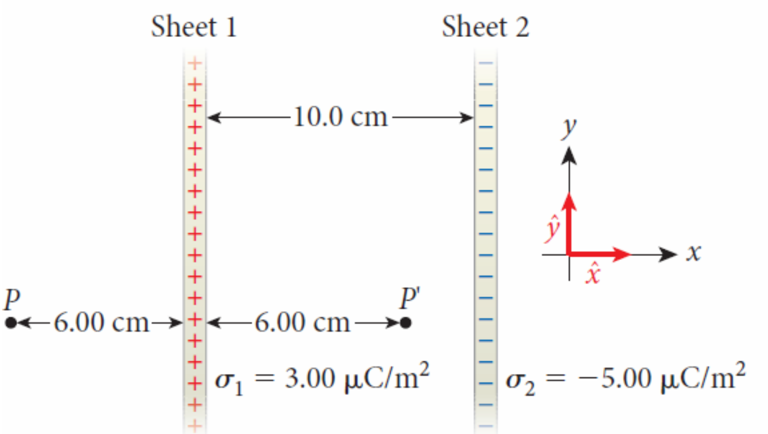


Fig. 9.1

(a) Electric field due to sheet 1, E_1,

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E_1 = \frac{\sigma_1}{2\varepsilon_0}$$

$$= \frac{3.00\times10^{-6}}{2\left(8.85\times10^{-12}\right)}$$

$$= 1.69\times10^{5}\ \text{N C}^{-1}$$

Direction of field is away from sheet 1.

Electric field due to sheet 2, E_2,

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E_2 = \frac{\sigma_2}{2\varepsilon_0}$$

$$= \frac{5.00\times10^{-6}}{2\left(8.85\times10^{-12}\right)}$$

$$= 2.82\times10^{5}\ \text{N C}^{-1}$$

Direction of field is towards sheet 2.

At P, electric field, E_P,

$$E_P = (-1.69 + 2.82)\times10^{5}$$

$$= 1.13\times10^{5}\ \text{N C}^{-1}$$

At P', electric field, $E_{P'}$,

$$E_{P'} = (1.69 + 2.82)\times10^{5}$$

$$= 4.52\times10^{5}\ \text{N C}^{-1}$$

Electric field is 1.13×10^5 N C^{-1} to the right. Electric field is 4.52×10^5 N C^{-1} to the right.

9 A non-conducting sphere has a uniform volume charge density ρ. Let $\vec{r}$ be the vector from the centre of the sphere to a general point *P* within the sphere.

(a) Show that the electric field at *P* is given by $\dfrac{\rho\vec{r}}{3\varepsilon_0}$. (Note that the result is independent of the radius of the sphere.)

(b) A spherical cavity is hollowed out of the sphere, as shown in Fig. 11.1.

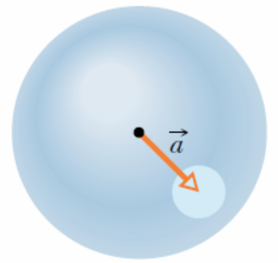


Fig. 11.1

Using superposition concepts, show that the electric field at all points within the cavity is uniform and equal to $\dfrac{\rho\vec{a}}{3\varepsilon_0}$, where $\vec{a}$ is the position vector from the centre of the sphere to the centre of the cavity. (Note that this result is independent of the radius of the sphere and the radius of the cavity)

(a)

$$q = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi\rho r^3$$

$$\int_A \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{\left(\frac{4}{3}\pi\rho r^3\right)}{\varepsilon_0}$$

$$E = \frac{\rho r}{3\varepsilon_0}$$

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0} \text{ (shown)}$$

(b) Let the position vector of a point, P, in cavity with respect to the centre of the sphere be $\vec{x}$. From (a), electric field due to the non-conducting charged sphere at P, $\vec{E}_1$

$$\vec{E}_1 = \frac{\rho\vec{x}}{3\varepsilon_0}$$

Let the position vector of a point in cavity with respect to the centre of the cavity be $\vec{y}$. From (a), electric field due to the cavity, $\vec{E}_2$

$$\vec{E}_2 = -\frac{\rho\vec{y}}{3\varepsilon_0}$$

Hence the net electric field at P,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$= \frac{\rho}{3\varepsilon_0}(\vec{x} - \vec{y})$$
$$= \frac{\rho\vec{a}}{3\varepsilon_0} \quad (\because \vec{a} = \vec{x} - \vec{y})(\text{shown})$$

10 Suppose the charge density of a solid sphere in is given by $\rho = \alpha r^2$ where α is a constant.

(a) Determine α in terms of the total charge Q on the sphere and its radius r_0.

(b) Determine the electric field as a function of r inside the sphere.

$$dq = \rho dV$$
$$= (\alpha r^2)(4\pi r^2)dr$$
$$Q = 4\alpha\pi\int_0^{r_0} r^4 dr$$
$$= \frac{4\alpha\pi}{5}\left[r^5\right]_0^{r_0}$$
$$= \frac{4\alpha\pi r_0^5}{5}$$

(a)
$$\alpha = \frac{5Q}{4\pi r_0^5}$$

$$q = 4\alpha\pi\int_0^{r} r^4 dr$$
$$= \frac{4\alpha\pi}{5}\left[r^5\right]_0^{r_0}$$
$$= \frac{4\alpha\pi r^5}{5}$$
$$= \frac{4}{5}\left(\frac{5Q}{4\pi r_0^5}\right)\pi r^5$$

(b)
$$= \left(\frac{r}{r_0}\right)^5 Q$$

By Gauss's law,

$$\int_A \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$
$$E(4\pi r^2) = \left(\frac{r}{r_0}\right)^5\left(\frac{Q}{\varepsilon_0}\right)$$
$$E = \frac{k_e Q}{r_0^5} r^3$$

Electric and Magnetic Fields tutorial SOLUTIONS

11 State a similarity and two differences between the application of Gauss's and Ampere's laws.

Similarity: Ampere's law, like Gauss's law, is always a valid statement but their application is limited to systems with a high degree of symmetry and is useful for finding the field around the system.

Differences:

Gauss's law	Ampere's law
Electric field	Magnetic field
Applies to any closed surface	Applies to any closed path
Relates the electric field on the surface to the net **electric charge** inside the surface	Relates the magnetic field on the path to the net **current** cutting through interior of the path
Component of the electric field **perpendicular** to the surface ($E_\perp$)	Component of the magnetic field **parallel** to the path ($B_\parallel$)
Used to determine electric field	Used to determine magnetic field

12 The current I within a wire that has a circular cross section of radius R is known to be distributed uniformly over that cross section.

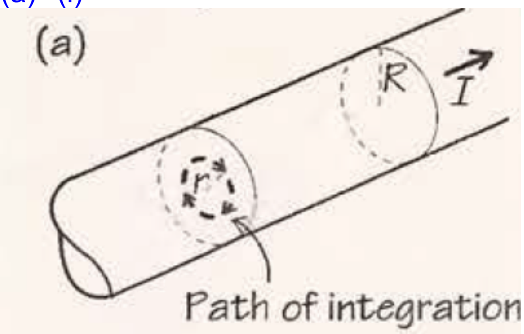
(a) Determine the magnetic field as a function of the distance r from the wire's axis where

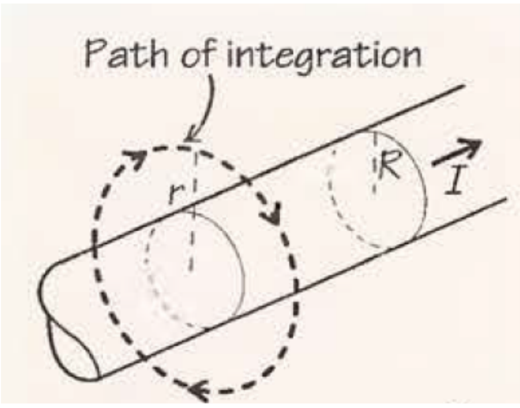
(i) $r < R$

(ii) $r > R$

(b) Sketch the $B - r$ graph.

(a) (i)





$$J = \frac{I}{A}$$

$$\frac{I'}{I} = \frac{A'}{A}$$

$$I' = \left(\frac{r}{R}\right)^2 I$$

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B(2\pi r) = \mu_0 I'$$

$$B = \frac{\mu_0 \left(\frac{r}{R}\right)^2 I}{2\pi r}$$

$$= \frac{\mu_0 I}{2\pi R^2} r$$

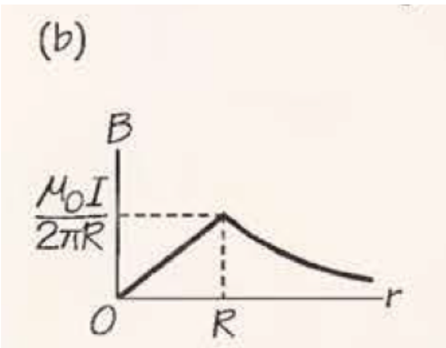
(ii)

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(b)

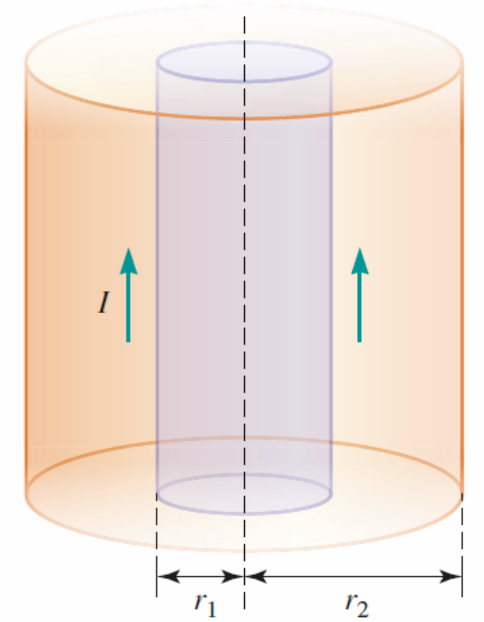


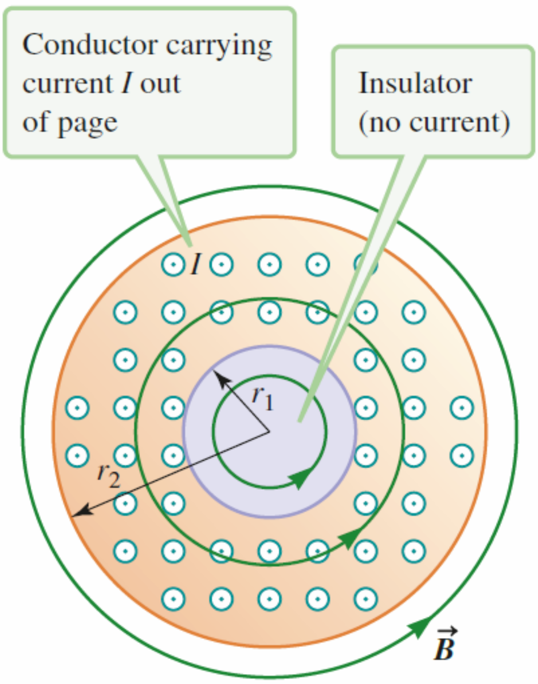
13 A very long, cylindrical wire has an insulating core surrounded by a cylindrical conductor carrying current I. The current is uniformly distributed in the conducting cylinder. The radius of the insulating core is r_1, and the outer radius of the conducting cylinder is r_2. Determine expressions for the magnetic flux density

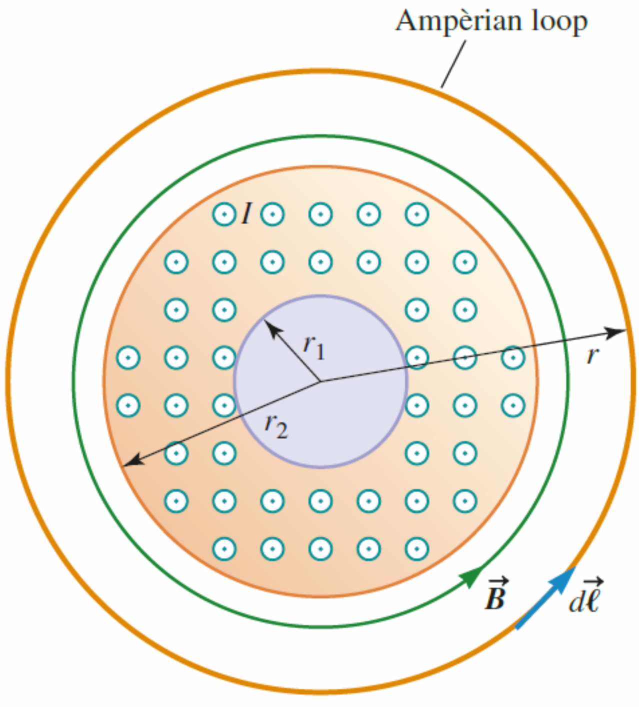
(a) outside the conducting cylinder ($r > r_2$),

(b) inside the conducting cylinder but outside the insulator ($r_1 < r < r_2$),

(c) and inside the insulator ($r < r_1$).







(a)

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(b)

$$J = \frac{I}{A} \quad (\text{where } J \text{ is the current density})$$

$$J = \frac{I}{\pi\left(r_2^2 - r_1^2\right)}$$

$$I' = JA'$$

$$= \frac{I}{\pi\left(r_2^2 - r_1^2\right)}\left[\pi\left(r^2 - r_1^2\right)\right]$$

$$= \frac{r^2 - r_1^2}{r_2^2 - r_1^2} I$$

By Ampere's law,

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B(2\pi r) = \mu_0\left(\frac{r^2 - r_1^2}{r_2^2 - r_1^2} I\right)$$

$$B = \frac{\mu_0 I\left(r^2 - r_1^2\right)}{2\pi r\left(r_2^2 - r_1^2\right)}$$

Checking, substitute $r = r_2$ into the above equation,

$$B = \frac{\mu_0 I\left(r^2 - r_1^2\right)}{2\pi r\left(r_2^2 - r_1^2\right)}$$
$$= \frac{\mu_0 I}{2\pi r_2}$$

which is the same as the expression derived in part (a) for $r = r_2$.

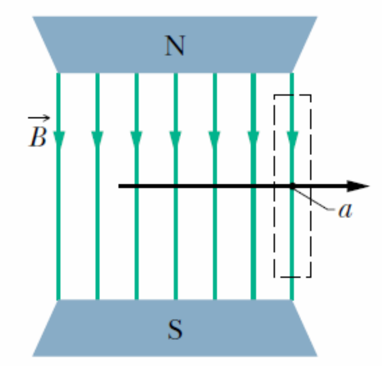
(c) By Ampere's law,

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$
$$B(2\pi r) = 0 \quad (\because I = 0 \text{ as there is no enclosed current})$$

Checking, substitute $r = r_1$ into the equation derived in part (b),

$$B = \frac{\mu_0 I\left(r^2 - r_1^2\right)}{2\pi r\left(r_2^2 - r_1^2\right)}$$
$$= 0$$

14 Show that a uniform magnetic field cannot drop abruptly to zero.



Let h be the height of the loop. By Ampere's law,

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$
$$Bh = 0 \quad (\because \text{the loop does not enclose any current})$$
$$B = 0$$

which is a contradiction as the magnetic field between the magnets is not zero. In actual magnets "fringing" of the magnetic field lines always occurs, which means that approaches zero in a gradual manner.

15 A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of magnitude 0.800 A. Determine the magnitude of the magnetic field inside the toroid at

(a) the inner radius and

(b) the outer radius of the toroid.

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$
$$B(2\pi r) = \mu_0 N i$$

(a) $$B = \frac{\mu_0 N i}{2\pi r}$$

At the inner radius, r_i,

$$B_i = \frac{\mu_0 N i}{2\pi r_i}$$
$$= \frac{\left(4\pi \times 10^{-7}\right)(500)(0.800)}{2\pi\left(15.0 \times 10^{-2}\right)}$$
$$= 5.33 \times 10^{-4}\ \text{T}$$

(b) At the outer radius, r_o,

$$B_o = \frac{\mu_0 N i}{2\pi r_o}$$
$$= \frac{\left(4\pi \times 10^{-7}\right)(500)(0.800)}{2\pi\left[(15.0 + 5.0) \times 10^{-2}\right]}$$
$$= 4.00 \times 10^{-4}\ \text{T}$$

16 The current density in a cylindrical conductor of radius R varies as $J(r) = \dfrac{J_0 r}{R}$ (in the region from zero to R).

(a) Express the magnitude of the magnetic field, B, in the regions $r < R$ and $r > R$.

(b) Sketch the graph of B against r.

$$dI = J dA$$
$$I = \int_0^r \frac{J_0 r}{R}(2\pi r)\,dr$$
$$= \frac{2\pi J_0}{R}\int_0^r r^2 dr$$

(a) $$= \frac{2\pi J_0 r^3}{3R}$$

For $r < R$,

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$
$$B(2\pi r) = \mu_0\left(\frac{2\pi J_0 r^3}{3R}\right)$$
$$B = \frac{\mu_0 J_0 r^3}{3R}$$

B is proportional to r^3.

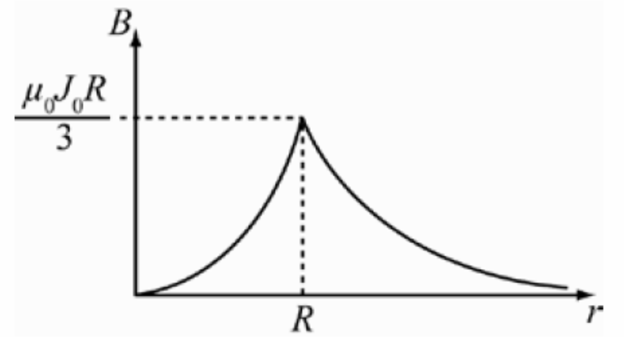
For $r > R$,

$$I = \frac{2\pi J_0 r^3}{3R}$$
$$= \frac{2\pi J_0 R^2}{3} \quad (\because r = R)$$

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I$$
$$B(2\pi r) = \mu_0\left(\frac{2\pi J_0 R^2}{3}\right)$$
$$B = \frac{\mu_0 J R^2}{3r}$$

B is inversely proportional to *r*.

(b)



17 A current loop, carrying an amount of current of 5.0 A, is in the shape of a right triangle with sides 30 cm, 40 cm, and 50 cm. The loop is in a uniform magnetic field of magnitude 80 mT whose direction is parallel to the current in the 50 cm side of the loop. Calculate the magnitude of the

(a) magnetic dipole moment of the loop,

(b) torque on the loop.

$$\begin{aligned}\mu &= NIA \\ &= IA \quad (\text{Q}\, N = 1) \\ &= \frac{1}{2} Ibh \\ &= \frac{1}{2}(5.0)(30\times10^{-2})(40\times10^{-2}) \\ \text{(a)} \quad &= 0.30 \text{ A m}^{-2}\end{aligned}$$

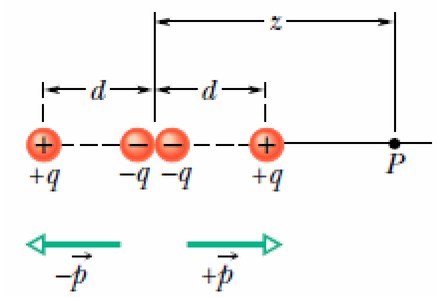
$$\begin{aligned}\tau &= \mu B \sin\theta \\ &= \mu B \quad (\text{Q} \sin\theta = 1) \\ &= 0.30(80\times10^{-3}) \\ \text{(b)} \quad &= 0.024 \text{ N m}\end{aligned}$$

18 Calculate the magnitude of the force, due to an electric dipole of dipole moment 3.6 × 10^{-29} C m, on an electron 25 nm from the centre of the dipole, along the dipole axis. Assume that this distance is large relative to the dipole's charge separation.

$$E = \frac{p}{2\pi\varepsilon_0 d^3}$$

$$\begin{aligned}F &= eE \\ &= \frac{pe}{2\pi\varepsilon_0 d^3} \\ &= \frac{(3.6\times10^{-29})(1.60\times10^{-19})}{2\pi(8.85\times10^{-12})(25\times10^{-9})^3} \\ &= 6.6\times10^{-15} \text{ N}\end{aligned}$$

19 Figure below shows an electric quadrupole. It consists of two dipoles with dipole moments that are equal in magnitude but opposite in direction. Determine the magnitude of $\vec{E}$ on the axis of the quadrupole at a point *P* a distance *z* from its centre. Assume $z >> d$.



$$
\begin{aligned}
\vec{E} &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \\
&= E_1 - E_2 - E_3 + E_4 \\
&= \frac{q}{4\pi\varepsilon_0 (z+d)^2} - \frac{q}{4\pi\varepsilon_0 z^2} - \frac{q}{4\pi\varepsilon_0 z^2} + \frac{q}{4\pi\varepsilon_0 (z-d)^2} \\
&= \frac{q}{4\pi\varepsilon_0}\left[(z+d)^{-2} - \frac{2}{z^2} + (z-d)^{-2}\right] \\
&= \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1+\frac{d}{z}\right)^{-2} - 2 + \left(1-\frac{d}{z}\right)^{-2}\right] \\
&= \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1-\frac{2d}{z}+\frac{3d^2}{z^2}\right) - 2 + \left(1+\frac{2d}{z}+\frac{3d^2}{z^2}\right)\right] + \ldots \\
&\approx \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{6d^2}{z^2}\right) \\
&= \frac{3qd^2}{2\pi\varepsilon_0 z^4}
\end{aligned}
$$

20 An electric dipole consists of charges $2e$ and $-2e$ separated by 0.78 nm. It is in an electric field of strength 3.4×10^6 N C^{-1}. Calculate the magnitude of the torque on the dipole when the dipole moment is

(a) parallel to,
(b) perpendicular to, and
(c) antiparallel to the electric field.

$$
\begin{aligned}
\vec{\tau} &= \vec{p} \times \vec{E} \\
&= pE\sin\theta \\
&= (qL)E\sin\theta \\
&= 2eLE\sin\theta
\end{aligned}
$$

(a) $= 0 \text{ N m} \quad (\because \theta = 0)$

$$
\begin{aligned}
\vec{\tau} &= \vec{p} \times \vec{E} \\
&= pE\sin\theta \\
&= (qL)E\sin\theta \\
&= 2eLE\sin\theta \\
&= 2(1.60\times10^{-19})(0.78\times10^{-9})(3.4\times10^{6})\sin\frac{\pi}{2}
\end{aligned}
$$

(b) $= 8.5\times10^{-22}$ N m

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$= pE\sin\theta$$
$$= (qL)E\sin\theta$$
$$= 2eLE\sin\theta$$

(c) $= 0 \text{ N m } (\because \theta = \pi)$

21 Sketch the graph of

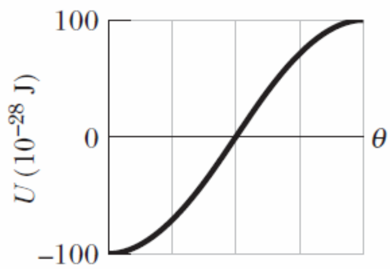
(a) τ against θ

(b) U against θ

of an electric dipole placed in a uniform electric field, where τ is the torque, U is the electric potential energy and θ is the angle between the dipole moment and the field.

Sine graph and negative cosine graph (until $\theta = \pi$).

22 Figure below shows the potential energy U of an electric dipole against the angle θ between a uniform electric field $\vec{E}$ of magnitude 20 N C^{-1} and the dipole moment $\vec{p}$.



Calculate is the magnitude of $\vec{p}$.

From graph, when $\theta = 0$, $U = -100 \times 10^{-28}$ J,

$$U = -\vec{p}.\vec{E}$$
$$= -pE\cos\theta$$
$$p = -\frac{U}{E\cos\theta}$$
$$= -\frac{(-100 \times 10^{-28})}{20\cos 0}$$
$$= 5.0 \times 10^{-28} \text{ C m}$$

23 Determine the frequency of oscillation of an electric dipole, of dipole moment $\vec{p}$ and moment of inertia I, for small amplitudes of oscillation about its equilibrium position in a uniform electric field $\vec{E}$.

By Newton's second law for rotation,

$$\vec{\tau} = I\vec{\alpha}$$
$$-(pE\sin\theta)\hat{\theta} = I\vec{\alpha} \quad (\because \vec{\alpha} \text{ is in the opposite direction of increasing } \theta)$$
$$\vec{\alpha} = -\frac{pE}{I}\vec{\theta} \quad (\because \sin\theta \approx \theta \text{ for small amplitudes of oscillation})$$
$$\omega^2 = \frac{pE}{I}$$
$$f = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}$$

21 Sketch the graph of

(a) τ against θ

(b) U against θ

of an electric dipole placed in a uniform electric field, where τ is the torque, U is the electric potential energy and θ is the angle between the dipole moment and the field.

Sine graph and negative cosine graph (until $\theta = \pi$).

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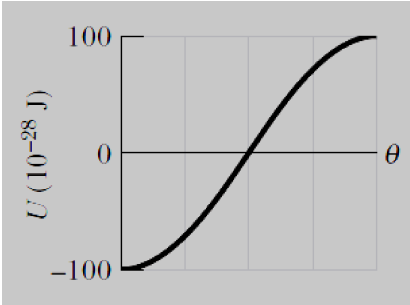


Calculate is the magnitude of $\vec{p}$.

From graph, when $\theta = 0$, $U = -100 \times 10^{-28}$ J,

$$\begin{aligned} U &= -\vec{p}\cdot\vec{E} \\ &= -pE\cos\theta \end{aligned}$$

$$\begin{aligned} p &= -\frac{U}{E\cos\theta} \\ &= -\frac{\left(-100\times10^{-28}\right)}{20\cos 0} \\ &= 5.0\times10^{-28}\ \text{C m} \end{aligned}$$

23 Determine the frequency of oscillation of an electric dipole, of dipole moment $\vec{p}$ and moment of inertia I, for small amplitudes of oscillation about its equilibrium position in a uniform electric field $\vec{E}$.

By Newton's second law for rotation,

$$\vec{\tau} = I\vec{\alpha}$$

$$-\left(pE\sin\theta\right)\hat{\theta} = I\vec{\alpha} \quad \left(\because \vec{\alpha} \text{ is in the opposite direction of increasing } \theta\right)$$

$$\vec{\alpha} = -\frac{pE}{I}\vec{\theta} \quad \left(\because \sin\theta \approx \theta \text{ for small amplitudes of oscillation}\right)$$

$$\omega^2 = \frac{pE}{I}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}$$

24 An electric dipole consists of an electron and a proton at a separation of 2.00 nm and it is in a uniform field of magnitude 3.00×10^6 N C^{-1}. Calculate the amount of energy required to turn an electric dipole from being lined up with the electric field to being lined up opposite the field.

By work-energy theorem,

$$E_i + W = E_f$$
$$U_i + W = U_f$$
$$-pE\cos\theta_i + W = -pE\cos\theta_f$$
$$W = pE(\cos\theta_i - \cos\theta_f)$$
$$= qLE(\cos\theta_i - \cos\theta_f)$$
$$= (1.60\times10^{-19})(2.00\times10^{-9})(3.00\times10^{6})(\cos 0 - \cos\pi)$$
$$= 1.92\times10^{-21}\ \text{J}$$

25 An infinite line of positive charge lies along the y axis, with charge density $\lambda = 2.00\,\mu\text{C m}^{-1}$. A dipole is placed with its center along the x axis at x = 25.0 cm. The dipole consists of two charges of magnitude 10.0 μC separated by 2.00 cm. The axis of the dipole makes an angle of 35.0° with the x axis, and the positive charge is further from the line of charge than the negative charge. Calculate the net force exerted on the dipole.

By Gauss's law,

$$\oint_A \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$
$$E(2\pi r)l = \frac{\lambda l}{\varepsilon_0}$$
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Taking left as positive,

$$F = q(E_- - E_+)$$
$$= \frac{q\lambda}{2\pi\varepsilon_0}\left(\frac{1}{r_-} - \frac{1}{r_+}\right) \quad \left(\text{where } r_- = x - \frac{L}{2}\cos\theta \text{ and } r_+ = x + \frac{L}{2}\cos\theta\right)$$
$$= \frac{(10.0\times10^{-6})(2.00\times10^{-6})}{2\pi(8.85\times10^{-12})}\left[\frac{1}{(25.0 - 1.00\cos 35.0°)10^{-2}} - \frac{1}{(25.0 + 1.00\cos 35.0°)10^{-2}}\right]$$
$$= 9.44\times10^{-2}\ \text{N}$$

26 A circular wire of radius 5.0 cm has a current of 3.0 A flowing in it. The wire is placed in a uniform magnetic field of 5.0 mT.

(a) Determine the maximum torque on the wire.

(b) Determine the range of the magnetic potential energy of the wire.

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$
$$\tau = \mu B\sin\theta$$
$$= \mu B \quad \left(\because \sin\theta = 1 \text{ i.e. } \theta = \frac{\pi}{2} \text{ for maximum } \tau\right)$$
$$= (IA)B$$
$$= I(\pi r^2)B$$
$$= 3.0(\pi)(5.0\times10^{-2})^2(5.0\times10^{-3})$$

(a) $= 1.2\times10^{-4}\ \text{N m}$

$$U = -\vec{\mu} \cdot \vec{B}$$
$$= -\mu B \cos\theta$$

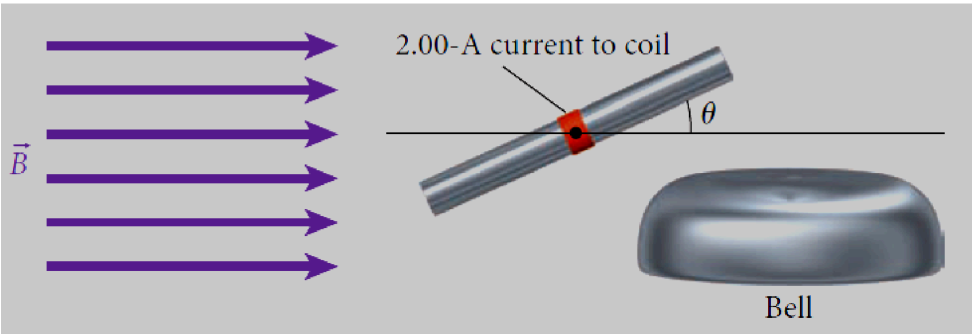
$U_{min} = -\mu B$ (U is minimum when $\theta = 0$)
$$= -IAB$$
$$= -3.0\pi\left(5.0\times10^{-2}\right)^2\left(5.0\times10^{-3}\right)$$
(b) $= -1.2\times10^{-4}$ J

$U_{max} = \mu B$ (U is maximum when $\theta = \pi$)
$$= IAB$$
$$= 3.0\pi\left(5.0\times10^{-2}\right)^2\left(5.0\times10^{-3}\right)$$
$$= 1.2\times10^{-4} \text{ J}$$

$$-1.2\times10^{-4} \text{ J} \le U \le 1.2\times10^{-4} \text{ J}$$

$$\Delta U = 1.2 - (-1.2)$$
$$= 2.4\times10^{-4} \text{ J}$$

27 An electromagnetic doorbell has 70 turns of wire around a long, thin rod, as shown. The rod has mass 30.0 g, length 8.00 cm, and a cross-sectional area of 0.200 cm^2. The rod is free to move and is pivoted about an axis through its centre, which coincides with the centre of the coil. The rod is initially at rest when θ = 25.0° with the horizontal. When θ = 0.00°, the rod strikes a bell. A uniform magnetic field of 9.00 × 10^{-2} T is directed at an angle θ = 0.00°. Given that a current of 2.00 A is in the coil, calculate



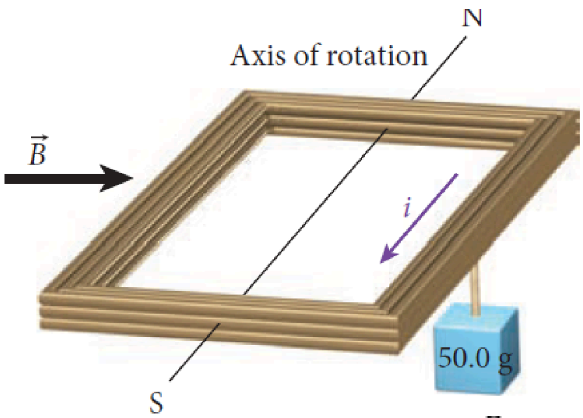
(a) the torque on the rod when θ = 25.0°
(b) the angular velocity of the rod when it strikes the bell.

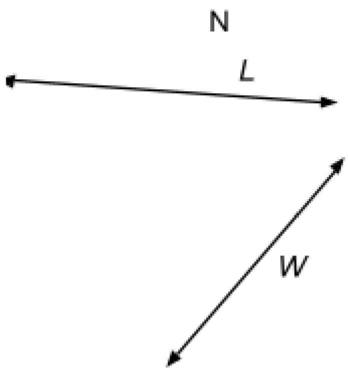
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$= \mu B \sin\theta$$
$$= NiAB \sin\theta$$
$$= 70(2.00)\left(0.200\times10^{-4}\right)\left(9.00\times10^{-2}\right)\sin 25.0°$$
(a) $= 1.06\times10^{-4}$ N m

(b) By law of conservation of energy,

$$E_i = E_f$$
$$U_i + K_i = U_f + K_f$$
$$-\mu B\cos\theta_i = -\mu B\cos\theta_f + \frac{1}{2}I\omega^2 \quad (\because K_i = 0)$$
$$\omega^2 = \frac{2\left[\mu B\left(\cos\theta_f - \cos\theta_i\right)\right]}{I}$$
$$= \frac{2\left[NiAB\left(\cos\theta_f - \cos\theta_i\right)\right]}{\left(\frac{1}{12}ml^2\right)}$$
$$\omega = \sqrt{\frac{24\left[NiAB\left(\cos\theta_f - \cos\theta_i\right)\right]}{ml^2}}$$
$$= \sqrt{\frac{24\left[(70)(2.00)\left(0.200\times10^{-4}\right)\left(9.00\times10^{-2}\right)\left(\cos 0.00° - \cos 25.0°\right)\right]}{\left(30.0\times10^{-3}\right)\left(8.00\times10^{-2}\right)^2}}$$
$$= 1.72 \text{ rad s}^{-1}$$

28 A 50-turn rectangular coil of wire with dimensions L = 10.0 cm by W = 20.0 cm lies in a horizontal plane, as shown in the figure. The axis of rotation of the coil is aligned north and south. It carries a current i = 1.00 A and is in a magnetic field pointing from west to east. A mass of 50.0 g hangs from one side of the coil. Determine the strength the magnetic field in order to have to keep the coil in the horizontal orientation.





Since the coil is in equilibrium,

$$\tau_{net} = 0$$
$$\mu B = mg\left(\frac{L}{2}\right) \quad (\because \sin\theta = 1)$$
$$NIAB = \frac{mgL}{2}$$
$$NI(LW)B = \frac{mgL}{2}$$
$$B = \frac{mg}{2NIW}$$
$$= \frac{\left(50.0\times10^{-3}\right)(9.81)}{2(50)(1.00)\left(20.0\times10^{-2}\right)}$$
$$= 0.0245 \text{ T}$$

29 In a highly simplified model of the atom, the hydrogen atom is assumed to consist of an electron moving with speed v in a circular orbit with radius r around a stationary proton. The centripetal force keeping the electron moving in a circle is the electric force between the proton and the electron. The radius of the orbit of the electron is $r = 5.29 \times 10^{-11}$ m.

(a) Derive an expression relating the magnetic dipole moment, $\vec{\mu}$, and the angular momentum $\vec{L}$ in terms of the mass of an electron, m, and the elementary charge, e.

(b) Calculate the magnitude of the orbital magnetic moment of the hydrogen atom.

(a) Magnitude of the magnetic dipole moment, μ,

$$\mu = IA$$
$$= \frac{e}{T}\left(\pi r^2\right)$$
$$= \frac{e}{\left(\frac{2\pi}{\omega}\right)}\left(\pi r^2\right)$$
$$= \frac{er^2\omega}{2}$$
$$\vec{\mu} = -\frac{er^2}{2}\vec{\omega}$$
$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{L} = mr^2\vec{\omega}$$

Hence $\vec{\mu} = -\frac{e}{2m}\vec{L}$.

(b) Electric force provides centripetal force, by Newton's second law of motion,

$$F_{net} = ma$$
$$F_c = ma_c$$
$$F_E = mr\omega^2$$
$$\frac{e^2}{4\pi\varepsilon_o r^2} = mr\omega^2$$
$$\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_o mr^3}}$$

$$\mu = IA$$
$$= \frac{e}{T}\left(\pi r^2\right)$$
$$= \frac{e}{\left(\frac{2\pi}{\omega}\right)}\left(\pi r^2\right)$$
$$= \frac{\omega e r^2}{2}$$
$$= \frac{er^2}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m r^3}}$$
$$= \frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m}}$$
$$= \frac{\left(1.60\times10^{-19}\right)^2}{4}\sqrt{\frac{5.29\times10^{-11}}{\pi\left(8.85\times10^{-12}\right)\left(9.11\times10^{-31}\right)}}$$
$$= 9.25\times10^{-24}\ \text{A m}^2$$

30 A magnetic dipole is in a uniform magnetic field.

(a) **(i)** Show that the magnetic dipole can oscillate as a torsional pendulum in simple harmonic motion when displaced from its equilibrium orientation and released.

(ii) State and explain an assumption made in **(i)**.

Given that the dipole is a compass needle (a light bar magnet) with a magnetic moment of magnitude μ and moment of inertia about its centre is I. It is mounted on a frictionless, vertical axle and it is placed in a horizontal magnetic field of magnitude *B.*

(b) **(i)** Determine the frequency of oscillation of the compass needle.

(ii) Explain how the compass needle can be conveniently used as an indicator of the magnitude of the external magnetic field.

(iii) If the frequency is 0.680 Hz in the Earth's local field, with a horizontal component of 39.2 μT, calculate the magnitude of a field parallel to the needle in which its frequency of oscillation is 4.90 Hz.

(a) (i) By Newton's second law for rotation,

$$\vec{\tau} = I\vec{\alpha}$$

$$-\mu B\sin\theta = I\alpha \quad (\because \text{direction of } \alpha \text{ is opposite the direction of increasing } \theta)$$

$$\alpha = -\frac{\mu B}{I}\theta \quad (\because \sin\theta \approx \theta \text{ for small angular displacements})$$

Since angular acceleration is proportional to the angular displacement measured from the equilibrium position and angular acceleration is opposite to angular displacement (or directed towards equilibrium position), the dipole is undergoing simple harmonic motion.

(ii) Angular displacement is small such that the small angle approximation $\sin\theta \approx \theta$ is valid.

(b) (i) Comparing $\alpha = -\frac{\mu B}{I}\theta$ with the defining equation of simple harmonic motion $\alpha = -\omega^2\theta$

$$\omega^2 = \frac{\mu B}{I}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$$

(ii) From (i), $f \propto \sqrt{B}$ as μ and I are constants, therefore the frequency of oscillation will be higher in a stronger magnetic field. Period, T, can be determined by timing (t seconds) multiple (N) oscillations using a stopwatch $\left(T = \frac{t}{N}\right)$ and frequency, f, can be calculated easily using the equation $f = \frac{1}{T}$.

$$f \propto \sqrt{B}$$

$$\frac{f'}{f} = \sqrt{\frac{B'}{B}}$$

$$\frac{B'}{B} = \left(\frac{f'}{f}\right)^2$$

$$B' = \left(\frac{f'}{f}\right)^2 B$$

$$= \left(\frac{4.90}{0.680}\right)^2 \left(39.2 \times 10^{-6}\right)$$

(iii) $$= 2.04 \text{ mT}$$