FRAMES OF REFERENCE (NON-RELATIVISTIC)

1 A car travels due east with a speed of 15.0 m s^{-1}. Raindrops are falling at a constant speed vertically with respect to the Earth. The traces of the rain on the side windows of the car make an angle of 60.0° with the vertical.

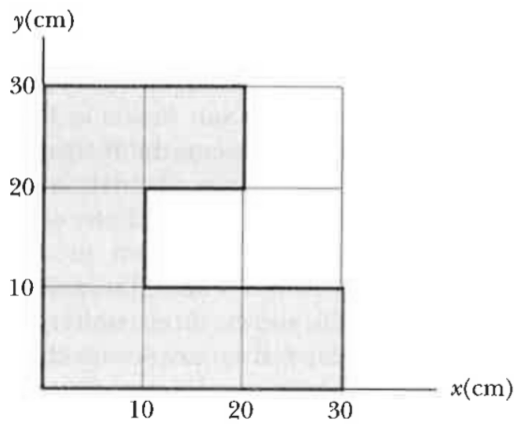
Determine the velocity of the raindrop with respect to

(a) the car.
(b) the Earth.

2 A bolt drops from the ceiling of a moving train car that is accelerating northward at a rate of 2.50 m s^{-2}.

(a) What is the acceleration of the bolt relative to the Earth?
(b) What is the acceleration of the bolt relative to the train?
(c) Describe the trajectory of the bolt as seen by an observer inside the train car.
(d) Describe the trajectory of the bolt as seen by an observer fixed on the Earth.

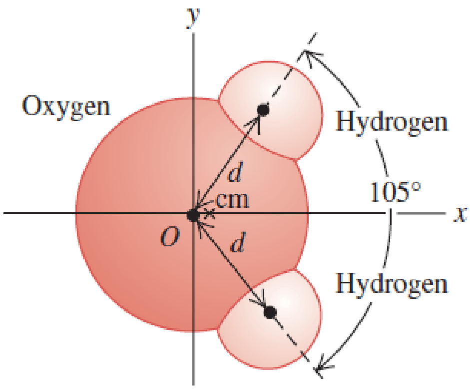
3 A uniform piece of steel sheet is shaped as below. Find the x and y coordinates of the centre of mass of the piece.



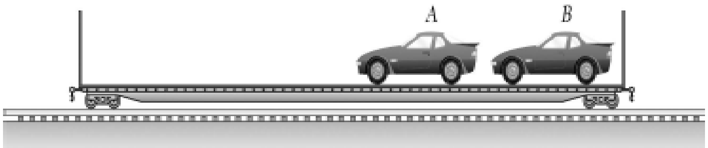
4 A stone is dropped at $t = 0$. A second stone, with twice the mass of the first, is dropped from the same point at $t = 100$ ms.

(a) How far below the release point is the center of mass of the 2 stones at $t = 300$ ms? (Assume neither stone has yet reached the ground.)
(b) How fast is the center of mass of the two-stone system moving at that time?

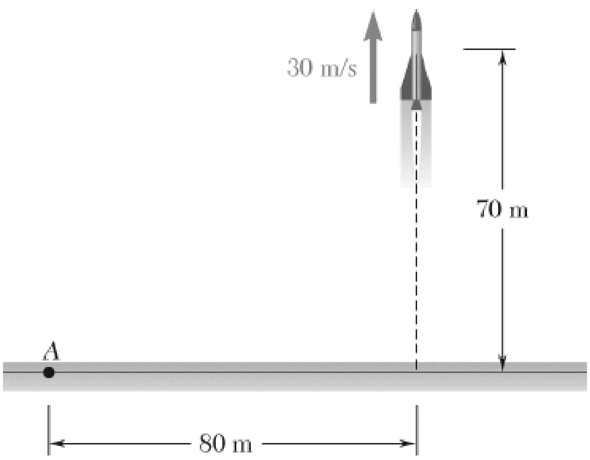
5 The figure shows a simple model of the structure of a water molecule. The separation between molecules d is 9.57×10^{-11} m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Determine the position of the centre of mass.



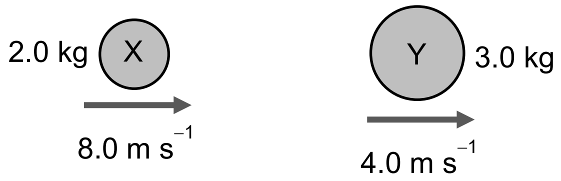
6 Car A of mass 1800 kg and car B of mass 1500 kg are at rest on a 22 000 kg stationary flatcar.
Assuming that the length the car is negligible compared to the length of the flatcar, determine the velocity of the flatcar when car A and B are moving at constant speeds of 2.5 m s^{-1} and 1.0 m s^{-1} relative to the flatcar respectively.



7 A 2.0 kg model rocket is launched vertically. When it reaches an altitude of 70 m with a speed of 30 m s^{-1}, it explodes into two parts m_A and m_B of masses 0.70 kg and 1.30 kg respectively. 6.0 s after the explosion, mass m_A is observed to strike the ground 80 m west of the launch point. Determine the position of mass m_B at that time.



8 Bodies X and Y of mass 2.0 kg and 3.0 kg respectively approach each other with speeds 8.0 m s^{-1} and 4.0 m s^{-1} respectively. Given that the two bodies collide head-on elastically, determine their speeds after the collision.



9 A ball of mass *3M* is held at rest at a height h above the ground. A smaller ball of mass *M* is held at rest directly above it. There is a small gap between the two balls, as shown in Fig. 9.1.

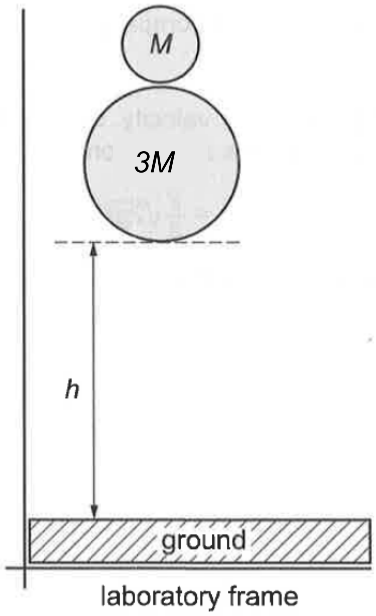


Fig. 9.1

The balls are released simultaneously and fall under gravity.

The ball of mass *3M* collides with the ground and rebounds.

A negligible time afterwards, the ball of mass *M* collides with the rebounding ball of mass *3M*, as shown in Fig. 9.2.

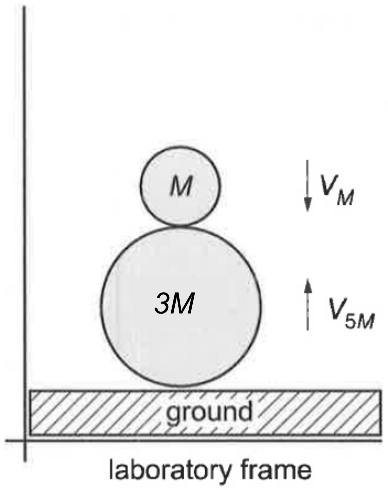


Fig. 9.2

You may assume that:

- all collisions are elastic
- the diameters of the balls are negligible compared to h
- air resistance is negligible

(a) (i) Show that the magnitude of the velocity v_{ZMF} of the zero momentum frame for the collision in Fig. 9.2 is given by the expression:

$$v_{ZMF} = \sqrt{\frac{gh}{2}}$$

where g is the acceleration of free fall.

(ii) Draw and label arrows on Fig. 9.3 to show:

- the velocity of the zero momentum frame relative to the laboratory frame
- the velocities, in terms of g and h, of the balls just before (left) and just after (right) they collide in the zero momentum frame.

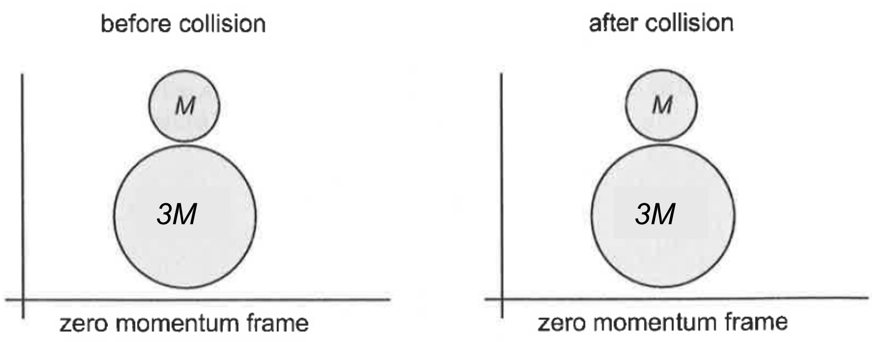


Fig. 9.3

(b) (i) Determine an expression for the height, in terms of h, which the ball of mass *3M* rebounds.

(ii) Use conservation of energy to determine an expression for the height, in terms of h, to which the ball of mass M rebounds.

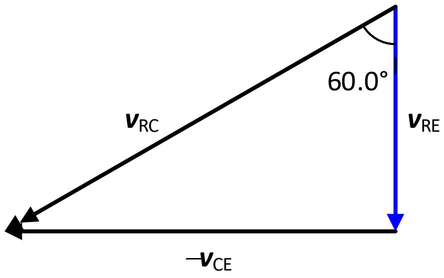
Answers:

1. 8.66 m s^{-1}, 17.3 m s^{-1}
2. **(a)** 10.1 m s^{-1}, **(c)** $\theta = 14.3°$
3. (11.7, 13.3) m
4. **(a)** $s_{CM} = 0.28$ m, **(b)** $v_{CM} = 2.29$ m s^{-1}
5. 6.5 x 10^{-12} m
6. 0.237 m s^{-1} to the right
7. 43 m east, 113 m above
8. 3.2 m s^{-1}, 7.2 m s^{-1}

Suggested Solutions to H3 Tutorial on Frames of Reference (Non-Relativistic)

1 The velocities are

v_{RE}: Raindrop relative to Earth
v_{CE}: Car relative to Earth
v_{RC}: Raindrop relative to Car



These vectors satisfy the vector triangle on the right, which says

$$\boldsymbol{v}_{RC} = \boldsymbol{v}_{RE} - \boldsymbol{v}_{CE}$$

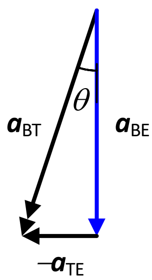
[Please try to derive this vector triangle. You can start from the position vectors.]
Notice that v_{CE} and v_{RE} are perpendicular to each other, one being vertical and the other horizontal.

(a) $\left|\boldsymbol{v}_{RE}\right| = \left|\boldsymbol{v}_{CE}\right| \tan 30.0 = 8.66 \text{ m s}^{-1}$

(b) $\left|\boldsymbol{v}_{RC}\right| = \dfrac{\left|\boldsymbol{v}_{CE}\right|}{\sin 60.0°} = 17.3 \text{ m s}^{-1}$

2 The problem provides a simple exploration of a non-inertial frame of reference where fictitious forces are required to explain the observation in the non-inertial frame of reference.

(a) Taking the derivative of the corresponding velocity equation (which you can derive),



$$\boldsymbol{a}_{BT} = \boldsymbol{a}_{BE} - \boldsymbol{a}_{TE}$$

Acceleration of bolt relative to Earth is always $\boldsymbol{g}$ (downwards).

$$\left|\boldsymbol{a}_{BE}\right| = g = 9.81 \text{ m s}^{-2}$$

which is perpendicular to a_{TE} (horizontal). Therefore,

$$\left|\boldsymbol{a}_{BT}\right| = \sqrt{\left|\boldsymbol{a}_{BE}\right|^2 - \left|\boldsymbol{a}_{TE}\right|^2} = \sqrt{9.81^2 + 2.50^2} = 10.1 \text{ m s}^{-2}$$

(c)

$$\theta = \tan^{-1}\frac{2.50}{9.81} = 14.3°$$

The bolt is seen to accelerate in a straight line at an angle of 14.3° to the vertical (towards rear of train). The trajectory is a straight line slanting diagonally downwards and backwards (towards the South).
Explanation: To an observer inside the train, the bolt is initially at rest on the ceiling. Once it drops, it is subject to the constant relative acceleration calculated in part (b) (downwards and backwards). Because the bolt starts with zero initial velocity relative to the observer and undergoes constant acceleration, it travels in a straight line.

(d) The bolt accelerates vertically downwards at a rate of 9.81 m s^{-2}. The trajectory is a parabola curving downwards and forwards (towards the North), characteristic of standard projectile motion.
Explanation: To an observer on the ground, the bolt has an initial horizontal velocity. At the exact moment it drops, it is moving North at the same speed as the train. Once it drops, a constant vertical force (gravity) pulls it down, while its horizontal velocity remains constant (due to inertia).

3

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + ... + m_6x_6}{m_1 + m_2 + ... + m_6} = \frac{3(1)(5) + 2(1)(15) + 1(1)(25)}{6} = 11.7\text{ m}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + ... + m_6y_6}{m_1 + m_2 + ... + m_6} = \frac{3(1)(5) + 1(1)(15) + 2(1)(25)}{6} = 13.3\text{ m}$$

4 **(a)**

$$s = ut + \frac{1}{2}at^2$$

$$s_1 = \frac{1}{2}(9.81)\left(300\times10^{-3}\right)^2 = 0.44\text{ m}$$

$$s_2 = \frac{1}{2}(9.81)\left(200\times10^{-3}\right)^2 = 0.20\text{ m}$$

$$s_{CM} = \frac{m_1s_1 + m_2s_2}{m_1 + m_2} = \frac{m(0.44) + 2m(0.20)}{m + 2m} = 0.28\text{ m}$$

(b)

$$v = u + at$$

$$v_1 = (9.81)\left(300\times10^{-3}\right) = 2.94\text{ m s}^{-1}$$

$$v_2 = (9.81)\left(200\times10^{-3}\right) = 1.96\text{ m s}^{-1}$$

$$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{m(2.94) + 2m(1.96)}{m + 2m} = 2.29\text{ m s}^{-1}$$

5

$$x_{CM} = \frac{(16.0u)(0) + (1.0u)(d\cos 52.5°) + (1.0u)(d\cos 52.5°)}{16.0u + 1.0u + 1.0u}$$

$$= \frac{2(9.57\times10^{-11})(\cos 52.5°)}{18.0}$$

$$= 6.47\times10^{-12}\text{ m}$$

$y_{CM} = 0$ by symmetry

6

$v_{CM} = 0$ since $\sum F_{ext} = 0$

Taking to the right as positive,

$$0 = \frac{(1800)(-2.5) + (1500)(-1.0) + (22000)(v_f)}{1800 + 1500 + 22000}$$

$v_f = 0.273\text{ m s}^{-1}$ to the right

7 At t = 0, x_{CM} = 0 m, y_{CM} = 70 m

Since $\sum F_{ext,x} = 0$, x_{cm} remains the same.

At t = 6.0 s,
x_{CM} = 0

$$0 = \frac{(0.70)(-80)+(1.30)(x_B)}{0.70+1.30}$$

x_B = 43.1 m to the east

Since $\sum F_{ext,y} \neq 0$, y_{cm} will accelerate in the direction of weight.

At t = 6.0 s,

$$y_{CM,f} - y_{CM,i} = u_x t + \frac{1}{2}gt^2$$

$$y_{CM,f} - 70 = (30)(6.0) + \frac{1}{2}(-9.81)(6.0)^2$$

$$y_{CM,f} = 73.42 \text{ m}$$

Applying $y_{CM,f} = \dfrac{m_A y_A + m_B + y_B}{m_A + m_B}$

$$73.42 = \frac{(0.70)(0.00)+(1.30)(y_B)}{(0.70+1.30)}$$

y_B = 113 m to the north

8 **Method 1 (H3 9814)**

$$v_{CM} = \frac{(2.0)(8.0)+(3.0)(4.0)}{2.0+3.0} = 5.6 \text{ m s}^{-1}$$

Using CM frame of reference,

$$u'_X = u_X - v_{CM} = 8.0 - 5.6 = 2.4 \text{ m s}^{-1}$$

$$u'_Y = u_Y - v_{CM} = 4.0 - 5.6 = -1.6 \text{ m s}^{-1}$$

Since this is a 1D head-on elastic collision, total momentum and total kinetic energy of the system is conserved.

$$v'_X = -u'_X = -2.4 \text{ m s}^{-1}$$

$$v'_Y = -u'_Y = 1.6 \text{ m s}^{-1}$$

Hence

$$v_X = v'_X + v_{CM} = -2.4 + 5.6 = 3.2 \text{ m s}^{-1}$$

$$v_Y = v'_Y + v_{CM} = 1.6 + 5.6 = 7.2 \text{ m s}^{-1}$$

Method 2 (H2 9478)
By conservation of momentum,

$$m_x u_x + m_Y u_Y = m_x v_x + m_Y v_Y$$

$$(2.0)(8.0) + (3.0)(4.0) = (2.0)v_X + (3.0)v_Y$$

$$28 = 2.0v_X + 3.0v_Y \quad ---(1)$$

Since the collision is elastic,

$$u_X - u_Y = v_Y - v_x$$

$$4.0 = -v_X + v_Y \quad ---(2)$$

Solving (1) & (2),

$v_X = 3.2 \text{ m s}^{-1}$ and $v_Y = 7.2 \text{ m s}^{-1}$

9 **(a)** **(i)** We are considering the situation of just before the ball-ball collision (shortly after the big ball has rebounded off the ground)

The big ball (3*M*) is moving upwards with speed *V* (it hits the ground at speed *V* and rebound elastically)

The small ball (*M*) is moving downwards with speed *V*.

In the lab frame:

Taking upwards as positive,
Lab - frame velocities :

$$u_{3M,\text{lab}} = +V \quad , \quad u_{M,\text{lab}} = -V$$

$$v_{ZMF} = \frac{(3M)(V)+(M)(-V)}{3M+M} = \frac{1}{2}V$$

By conservation of energy :
Loss in GPE = Gain in KE

$$mgh = \frac{1}{2}mV^2$$

$$V = \sqrt{2gh}$$

$$\therefore v_{ZMF} = \frac{1}{2}V = \sqrt{\frac{gh}{2}} \text{ (shown)}$$

(ii) $v_{ball,ZMF} = v_{ball,lab} - v_{ZMF}$

Before collision (in ZMF) :

$$u_{3M,\text{ZMF}} = V - \frac{1}{2}V = \frac{1}{2}V \text{ (upwards)}$$

$$u_{M,\text{ZMF}} = -V - \frac{1}{2}V = -\frac{3}{2}V \text{ (downwards)}$$

Because collision is elastic and ZMF has zero total momentum, the velocities in the ZMF simple reverse sign on collision :

After collision (in ZMF) :

$$v_{3M,\text{ZMF}} = -\frac{1}{2}V \text{ (downwards)}$$

$$\Rightarrow v_{3M,\text{lab}} = v_{5M,\text{ZMF}} + v_{ZMF} = -\frac{1}{2}V + \frac{1}{2}V = 0$$

$$v_{M,\text{ZMF}} = \frac{3}{2}V \text{ (upwards)}$$

$$\Rightarrow v_{M,\text{lab}} = v_{M,\text{ZMF}} + v_{ZMF} = \frac{3}{2}V + \frac{1}{2}V = 2V \text{ (upwards)}$$

(b) **(i)** Height to which ball $3M$ rebounds. Its upward speed after the ball - ball collision
Using Principle of conservation of energy,
Loss in KE = Gain in GPE

$$\frac{1}{2}(3M)(0)^2 = (3M)(g)(H_{3M})$$

$$H_{3M} = 0$$

(ii) Height to which ball M rebounds. Its upward speed after the ball - ball collision

$v_{M,lab} = 2V$

Using Principle of conservation of energy,

Loss in KE = Gain in GPE

$$\frac{1}{2}(M)(2V)^2 = (M)(g)(H_M)$$

$$H_M = \frac{2V^2}{g} = \frac{2(2gh)}{g} = 4h$$