H3 Physics Topic 5 - Capacitors and Inductors (Non-Oscillatory)

1.

In the circuit of Fig. 1.1, switch S_1 is closed until the capacitor of capacitance C_1 is fully charged.

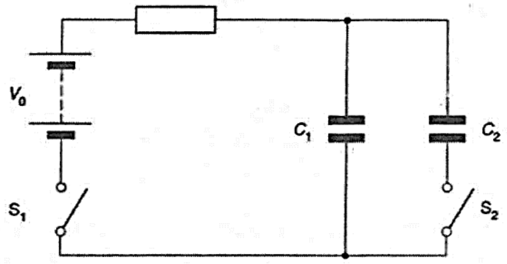


Fig. 1.1

Then, after S_1 is opened, switch S_2 is closed. On closing S_2, it is noticed that a spark develops between the contacts of the switch. Find an expression, in terms of C_1, C_2 and V_0, for the energy dissipated in this spark. [7]

Q1:

2.

A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d, as illustrated in Fig. 2.1.

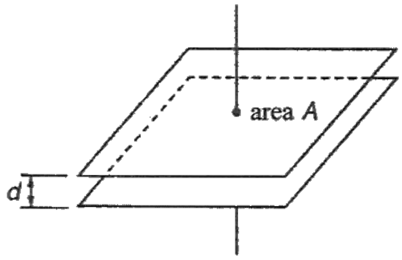


Fig. 2.1

When there is a vacuum between the plates, the capacitance C of the arrangement is given by

$$C = \varepsilon_0 A/d,$$

where ε_0 is a permittivity of free space.

If the space between the plates is completely filled with an insulator, the capacitance C' is given by

$$C = \varepsilon_0 \varepsilon_r A/d,$$

where ε_r is a constant characteristic of the material of the insulator, called the relative permittivity. The value of ε_r is 1 for air, and is greater than 1 for solid insulators.

A certain parallel-plate capacitor, with square plates of length L, has capacitance C when there is air between the plates. The capacitor is charged to a potential difference V_o and is then isolated. A student slowly inserts a slab of insulating material into the capacitor. The slab just fits between the plates and is of length L. The relative permittivity of the material of the slab is ε_r. At a certain instant the slab is a distance x into the gap, as shown in Fig. 2.2.

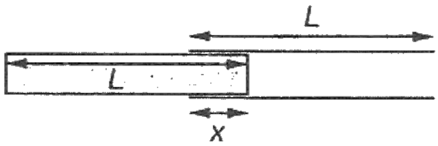


Fig. 2.2

(a) Show that the potential difference V_x between the plates is given by

$$V_x = LV_o / (L + x(\varepsilon_r - 1))$$

Explain your reasoning. [6]

(b) Find an expression for the electrical energy stored in the capacitor when the slab is in the position shown. Give your answer in terms of C, ε_r, L, V_o and x. Sketch a graph to show how the stored electrical energy depends on the distance x the slab is inserted. [5]

(c) The students stops the slab when it is half-way in, when $x = ½L$, and lets go of it. The slab accelerates from its position of rest.

(i) In which direction does the slab move? Why does it move in this direction? [2]

(ii) The mass of the slab is m. There is no friction between the slab and the plates. Find an expression for the maximum speed attained by the slab, in terms of C, ε_r, m and V_o. [5]

(iii) Describe the subsequent motion of the slab. [2]

Q2:

3.

(a) Fig. 3.1 shows a circuit containing a filament lamp in series with a battery and a capacitor. When the switch is closed, the lamp glows very briefly, but does not light permanently. This is not because the e.m.f. of the battery is too small.

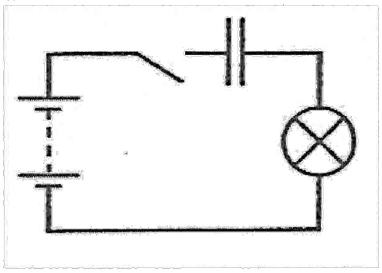


Fig. 3.1

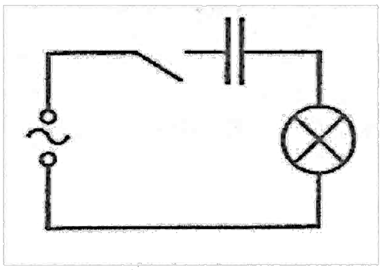


Fig. 3.2

Fig. 3.2 shows a similar circuit, but containing an a.c. supply instead of a battery. When the switch is closed, the lamp lights permanently.
Explain these observations. [5]

(b) Fig. 3.3 shows a circuit containing a resistor of resistance R and a capacitor of capacitance C connected in series with a battery of e.m.f. E and negligible internal resistance.

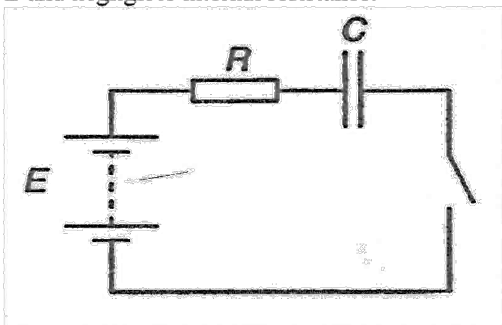


Fig. 3.3

The capacitor is initially uncharged. At time $t = 0$ the switch is closed. At a subsequent time t, the current in the circuit is I.

(i) Write down an equation representing Kirchhoff's second law at time t. Also, write down the equation linking the change dQ transferred in time dt by a current I. Hence show that the current I at time t is given by

$$I = (E/R)\, e^{-t/RC},$$ [6]

[*Hint*: $\int dx/(a-x) = -\ln(a-x) + b$, where b is a constant depending on the limits of integration.]

(ii) On the same axes of potential difference and time, sketch labeled graphs showing the variation with time t of the potential differences V_R across the resistor and V_C across the capacitor. [3]

(iii) Obtain an expression for the rate P_R at which energy is being dissipated in the resistor at time t. [2]

(iv) Obtain an expression for the rate P_C at which energy is being transferred to the capacitor at time t. [3]

(v) State the power supplied by the battery at time t. [1]

Q3:

4.

A capacitor is normally considered as consisting of two conducting plates separated by a vacuum or other insulator. When the capacitor is charged, there are equal and opposite charges on each plate. However, a single, electrically isolated conductor, of any shape, also has capacitance. When the conductor is charged, it carries a charge of one sign only.

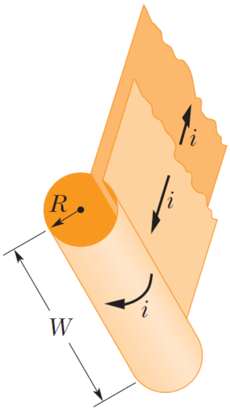
(a) Explain how an isolated conductor can have capacitance. Explain also how this conductor can operate as a capacitor with a charge of one sign only on it. [3]

(b) Estimate the capacitance of the Earth. Assume the Earth to be a conducting sphere of radius of 6.4×10^6 m. [2]

Q4:

5.

Figure shows a copper strip of width $W = 16.0$ cm that has been bent to form a shape that consists of a tube of radius $R = 1.8$ cm plus two parallel flat extensions. Current $i = 35$ mA is distributed uniformly across the width so that the tube is effectively a one-turn solenoid. Assume that the magnetic field outside the tube is negligible and the field inside the tube is uniform. What are (a) the magnetic field magnitude inside the tube and (b) the inductance of the tube (excluding the flat extensions)?



Q5:

6.

Two identical long wires of radius $a = 1.53$ mm are parallel and carry identical currents in opposite directions. Their center-to-center separation is $d = 14.2$ cm. Neglect the flux within the wires but consider the flux in the region between the wires. What is the inductance per unit length of the wires?

Q6:

7a

Inductors in series. Two inductors L_1 and L_2 are connected in series and are separated by a large distance so that the magnetic field of one cannot affect the other. Show that the equivalent inductance is given by

$$L_{eq} = L_1 + L_2.$$

7b

Inductors in parallel. Two inductors L_1 and L_2 are connected in parallel and separated by a large distance so that the magnetic field of one cannot affect the other. Show that the equivalent inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

Q7

8.

(a) The current in an RL circuit builds up to one-third of its steady-state value in 5.00 s. Find the inductive time constant.

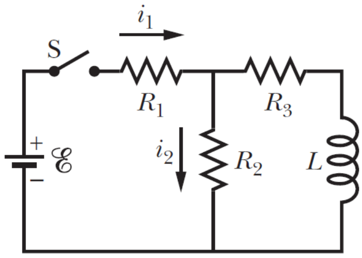
(b) The current in an RL circuit drops from 1.0 A to 10 mA in the first second following removal of the battery from the circuit. If L is 10 H, find the resistance R in the circuit.

(c) A solenoid having an inductance of 6.30 μH is connected in series with a 1.20 kΩ resistor. (i) If a 14.0 V battery is connected across the pair, how long will it take for the current through the resistor to reach 80.0% of its final value? (ii) What is the current through the resistor at time $t = 1.0\tau_L$?

Q8

9.

In Fig, $\mathscr{E} = 100$ V, $R_1 = 10.0\ \Omega$, $R_2 = 20.0\ \Omega$, $R_3 = 30.0\ \Omega$, and $L = 2.00$ H. Immediately after switch S is closed, what are (a) i_1 and (b) i_2? (Let currents in the indicated directions have positive values and currents in the opposite directions have negative values.) A long time later, what are (c) i_1 and (d) i_2? The switch is then reopened. Just then, what are (e) i_1 and (f) i_2? A long time later, what are (g) i_1 and (h) i_2?



Q9:

H3 Physics Topic 5 - Capacitors and Inductors (Non-Oscillatory)

Name: ______________________________ **Class:** __________

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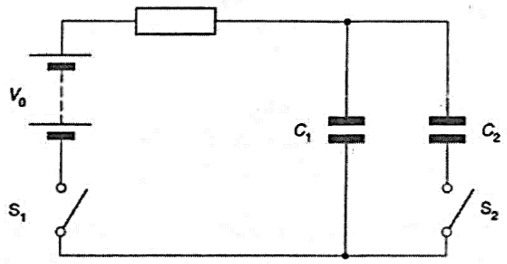

Fig. 1.1

Then, after S_1 is opened, switch S_2 is closed. On closing S_2, it is noticed that a spark develops between the contacts of the switch. Find an expression, in terms of C_1, C_2 and V_0, for the energy dissipated in this spark. [7]

Q1:

S_1 closed, C_1 charges to p.d. of V_0

initial energy stored in $C_1 = \frac{1}{2}C_1V_0^2$

initial charge stored in C_1: $Q_1 = C_1V_0$

when S_1 is opened and S_2 closed, total charge is conserved, but V changes

$$C_{\text{total}} = C_1 + C_2$$

$$V_{\text{new}} = \frac{Q_1}{C_{\text{total}}} = \frac{C_1V_0}{C_1 + C_2}$$

final energy stored in both capacitors

$$W = \frac{1}{2}C_{\text{total}}V_{\text{new}}^2$$

$$= \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_0}{C_1 + C_2}\right)^2$$

$$= \frac{1}{2}\left(\frac{(C_1V_0)^2}{C_1 + C_2}\right)$$

energy loss $= \frac{1}{2}C_1V_0^2 - \frac{1}{2}\left(\frac{(C_1V_0)^2}{C_1 + C_2}\right)$

$$= \frac{1}{2}(C_1V_0)^2\left(\frac{1}{C_1} - \frac{1}{C_1 + C_2}\right)$$

$$= \frac{C_1C_2V_0^2}{2(C_1 + C_2)}$$

2.

A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d, as illustrated in Fig. 2.1.

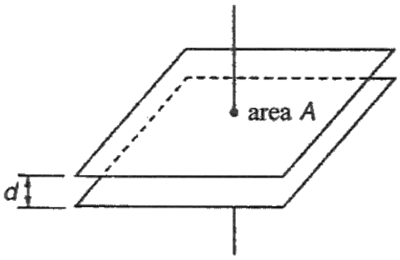

Fig. 2.1

When there is a vacuum between the plates, the capacitance C of the arrangement is given by

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where ε_0 is a permittivity of free space.
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A certain parallel-plate capacitor, with square plates of length L, has capacitance C when there is air between the plates. The capacitor is charged to a potential difference V_o and is then isolated. A student slowly inserts a slab of insulating material into the capacitor. The slab just fits between the plates and is of length L. The relative permittivity of the material of the slab is ε_r. At a certain instant the slab is a distance x into the gap, as shown in Fig. 2.2.

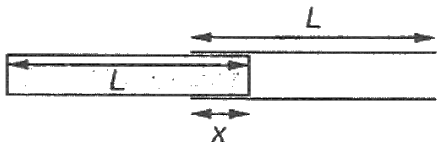


Fig. 2.2

(a) Show that the potential difference V_x between the plates is given by

$$V_x = LV_o / (L + x(\varepsilon_r - 1))$$

Explain your reasoning. [6]

(b) Find an expression for the electrical energy stored in the capacitor when the slab is in the position shown. Give your answer in terms of C, ε_r, L, V_o and x. Sketch a graph to show how the stored electrical energy depends on the distance x the slab is inserted. [5]

(c) The students stops the slab when it is half-way in, when $x = ½L$, and lets go of it. The slab accelerates from its position of rest.

(i) In which direction does the slab move? Why does it move in this direction? [2]

(ii) The mass of the slab is m. There is no friction between the slab and the plates. Find an expression for the maximum speed attained by the slab, in terms of C, ε_r, m and V_o. [5]

(iii) Describe the subsequent motion of the slab. [2]

Q2:

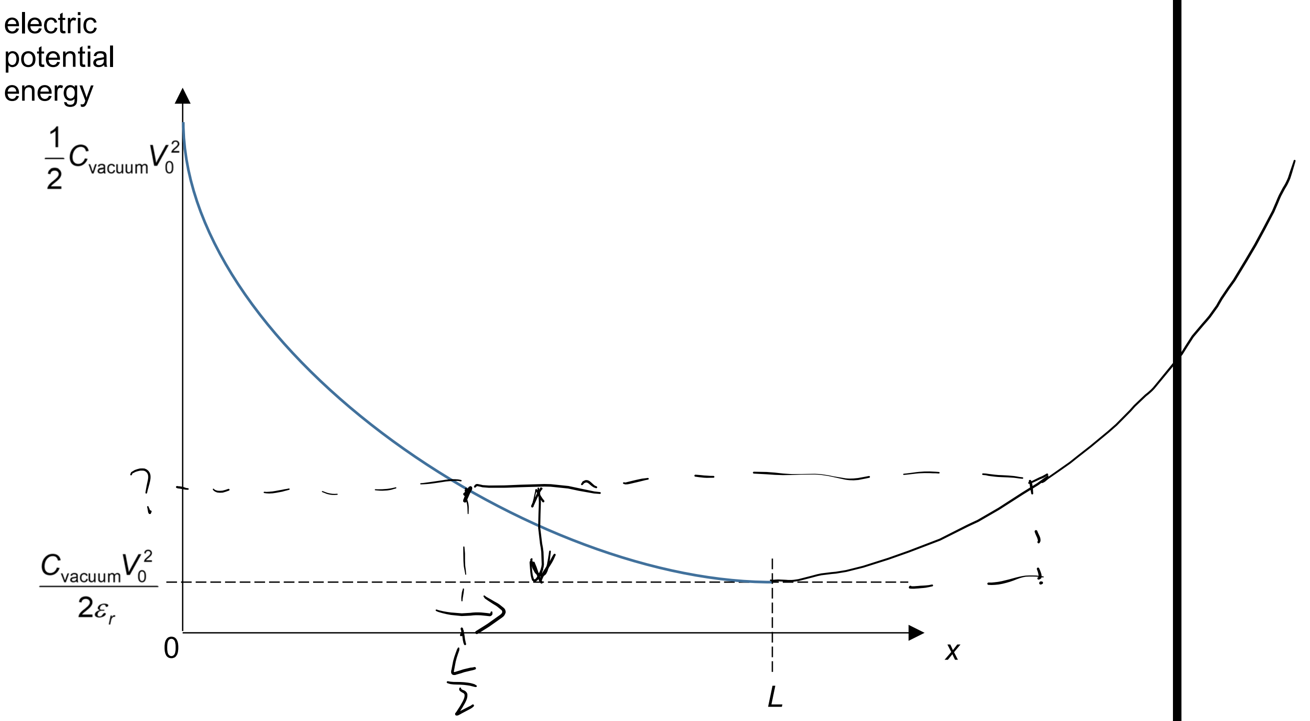
(a) Capacitor is charged without slab so $Q_{\text{total}} = C_{\text{vacuum}} V_0 = \dfrac{\varepsilon_0 A V_0}{d}$

With slab, system is slab plus w/o slab in parallel so $C_{\text{eff}} = \dfrac{\varepsilon_0 \varepsilon_r (xA)}{Ld} + \dfrac{\varepsilon_0 (L-x) A}{Ld}$

The total charge remains constant but the voltage V_x changes with how much the slab is inside the parallel plates so

$$V_x = \frac{Q_{\text{total}}}{C_{\text{eff}}} = \frac{\dfrac{\cancel{\varepsilon_0} \cancel{A} V_0}{\cancel{d}}}{\dfrac{\cancel{\varepsilon_0}\varepsilon_r (x\cancel{A})}{L\cancel{d}} + \dfrac{\cancel{\varepsilon_0}(L-x)\cancel{A}}{L\cancel{d}}} = \frac{V_0}{\dfrac{\varepsilon_r x}{L} + \dfrac{L-x}{L}} = \frac{LV_0}{L + x(\varepsilon_r - 1)}$$

(b) $U = \frac{1}{2} Q_{\text{total}} V_x = \frac{1}{2}(C_{\text{vacuum}} V_0)\left(\frac{LV_0}{L + x(\varepsilon_r - 1)}\right) = \frac{1}{2}\left(\frac{C_{\text{vacuum}} L V_0^2}{L + x(\varepsilon_r - 1)}\right)$



(c)(i) Slab accelerates to the right, further into capacitor, which decreases EPE. By PCE, slab gains KE

(ii) Max KE is when $x = L$ as EPE will be at minimum. For $x > L$, EPE rises again due to symmetry of capacitor geometry.

$$\text{gain in KE} = \text{loss in EPE}$$

$$E_{\text{K, f}} - 0 = U_{x=L/2} - U_{x=0}$$

$$\cancel{\frac{1}{2}} mv^2 = \cancel{\frac{1}{2}}\left(\frac{C_{\text{vacuum}} L V_0^2}{L + \left(\frac{L}{2}\right)(\varepsilon_r - 1)}\right) - \cancel{\frac{1}{2}}\left(\frac{C_{\text{vacuum}} V_0^2}{\varepsilon_r}\right)$$

$$mv^2 = \frac{C_{\text{vacuum}} V_0^2}{1 + \left(\frac{1}{2}\right)(\varepsilon_r - 1)} - \frac{C_{\text{vacuum}} V_0^2}{\varepsilon_r} = C_{\text{vacuum}} V_0^2\left(\frac{1}{1 + \frac{\varepsilon_r}{2} - \frac{1}{2}} - \frac{1}{\varepsilon_r}\right) = C_{\text{vacuum}} V_0^2\left(\frac{1}{\frac{\varepsilon_r}{2} + \frac{1}{2}} - \frac{1}{\varepsilon_r}\right)$$

$$= \frac{C_{\text{vacuum}} V_0^2}{\varepsilon_r}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)$$

$$v = V_0 \sqrt{\frac{C_{\text{vacuum}}}{m\varepsilon_r}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)}$$

(iii) when $x > L$, slab is attracted back into capacitor. motion is repeated, so slab oscillates about vertical axis of symmetry of capacitor.

3.

(a) Fig. 3.1 shows a circuit containing a filament lamp in series with a battery and a capacitor. When the switch is closed, the lamp glows very briefly, but does not light permanently. This is not because the e.m.f. of the battery is too small.

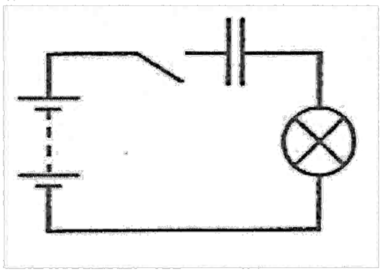


Fig. 3.1

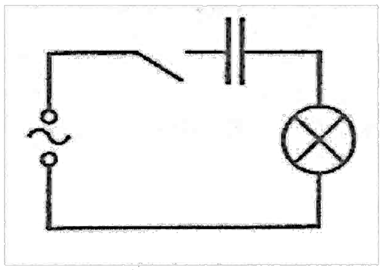


Fig. 3.2

Fig. 3.2 shows a similar circuit, but containing an a.c. supply instead of a battery. When the switch is closed, the lamp lights permanently.
Explain these observations. [5]

(b) Fig. 3.3 shows a circuit containing a resistor of resistance R and a capacitor of capacitance C connected in series with a battery of e.m.f. E and negligible internal resistance.

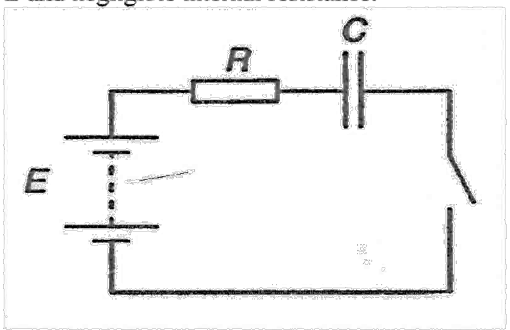


Fig. 3.3

The capacitor is initially uncharged. At time $t = 0$ the switch is closed. At a subsequent time t, the current in the circuit is I.

(i) Write down an equation representing Kirchhoff's second law at time t. Also, write down the equation linking the change dQ transferred in time dt by a current I. Hence show that the current I at time t is given by

$$I = (E/R)\, e^{-t/RC},$$ [6]

[*Hint*: $\int dx/(a-x) = -\ln(a-x) + b$, where b is a constant depending on the limits of integration.]

(ii) On the same axes of potential difference and time, sketch labeled graphs showing the variation with time t of the potential differences V_R across the resistor and V_C across the capacitor. [3]

(iii) Obtain an expression for the rate P_R at which energy is being dissipated in the resistor at time t. [2]

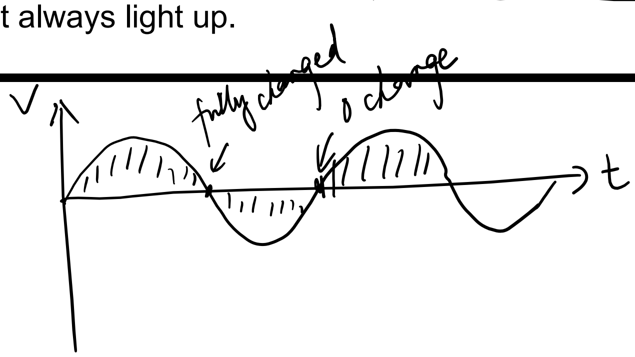
(iv) Obtain an expression for the rate P_C at which energy is being transferred to the capacitor at time t. [3]

(v) State the power supplied by the battery at time t. [1]

Q3:

(a) In a.c., the parallel plates of a capacitor are being charged and discharged periodically. Current is allowed to flow in alternate directions periodically. Hence, the bulb always lights up (there is always electrical energy being converted into heat and light).

In d.c., current stops flowing once the p.d. across the parallel plates of a capacitor is equal to the e.m.f.. The brief glow is from the non-steady flow of current during the charging process of the capacitor. There is no continuous flow of electrical current, hence the bulb does not always light up.



(b)(i) by Kirchoff's 2nd law:

$$E = V_R + V_C$$
$$= IR + \frac{Q}{C} = IR + \frac{\int I \, dt}{C}$$

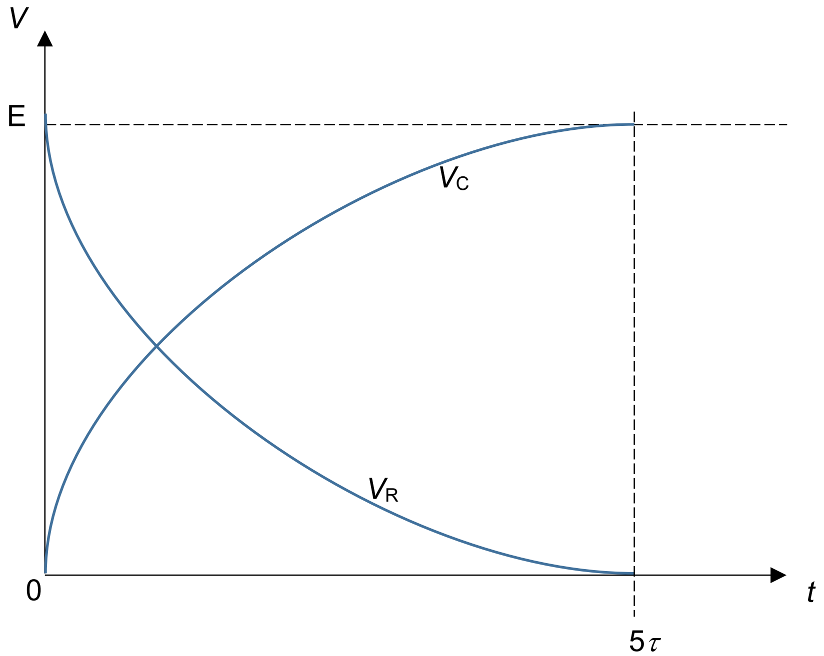
$$\frac{d}{dt}E = 0$$
$$= \left(\cancel{\frac{d}{dt}R}\right)I + R\frac{dI}{dt} + \frac{I}{C}$$

$$\int \frac{1}{I}\, dI = \int \frac{-1}{RC}\, dt$$
$$\ln I = (-1/RC)t + \ln k$$
$$I = ke^{(-1/RC)t}$$

when $t = 0$, $V_C = 0$ so $E = V_R \cancel{+V_C}$ and $I_0 = \frac{E}{R}$

$$I = \left(\frac{E}{R}\right)e^{(-1/RC)t}$$

(ii)



(iii) $$P_R = \frac{V_R^2}{R} = \left(\frac{E^2}{R}\right)e^{(-2/RC)t}$$

(iv) $$P_C = IV_C = \left[\frac{E}{R}\left(e^{(-1/RC)t}\right)\right]\left(E - Ee^{(-1/RC)t}\right)$$
$$= \left[\frac{E^2}{R}\left(e^{(-1/RC)t}\right)\right]\left(1 - e^{(-1/RC)t}\right)$$

(v) $$P_{\text{battery}} = IE = \left[\frac{E}{R}\left(e^{(-1/RC)t}\right)\right]E = \left[\frac{E^2}{R}\left(e^{(-1/RC)t}\right)\right]$$

4.

A capacitor is normally considered as consisting of two conducting plates separated by a vacuum or other insulator. When the capacitor is charged, there are equal and opposite charges on each plate. However, a single, electrically isolated conductor, of any shape, also has capacitance. When the conductor is charged, it carries a charge of one sign only.

(a) Explain how an isolated conductor can have capacitance. Explain also how this conductor can operate as a capacitor with a charge of one sign only on it. [3]

(b) Estimate the capacitance of the Earth. Assume the Earth to be a conducting sphere of radius of 6.4 x 10^6 m. [2]

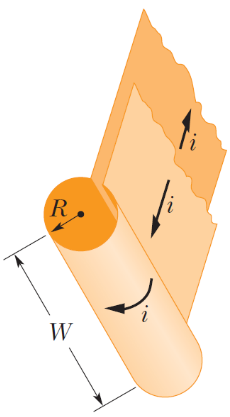
Q4:

(a) Charges of a single polarity are on conductors surface and charges of the opposite polarity can be regarded to be at infinity distance away.

(b) $$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0 r}} = 4\pi\varepsilon_0 r = 4\pi\left(8.85\times10^{-12}\right)\left(6.4\times10^{6}\right) = 7.1\times10^{-4}\ \text{F}$$

5.

Figure shows a copper strip of width W = 16.0 cm that has been bent to form a shape that consists of a tube of radius R = 1.8 cm plus two parallel flat extensions. Current i = 35 mA is distributed uniformly across the width so that the tube is effectively a one-turn solenoid. Assume that the magnetic field outside the tube is negligible and the field inside the tube is uniform. What are (a) the magnetic field magnitude inside the tube and (b) the inductance of the tube (excluding the flat extensions)?



Q5:

(a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is

$$B = \mu_0 n \Delta i = \mu_0\left(\frac{N}{W}\right)\left(\frac{i}{N}\right) = \frac{\mu_0 i}{W} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.035\text{A})}{0.16\text{m}} = 2.7\times10^{-7}\,\text{T}.$$

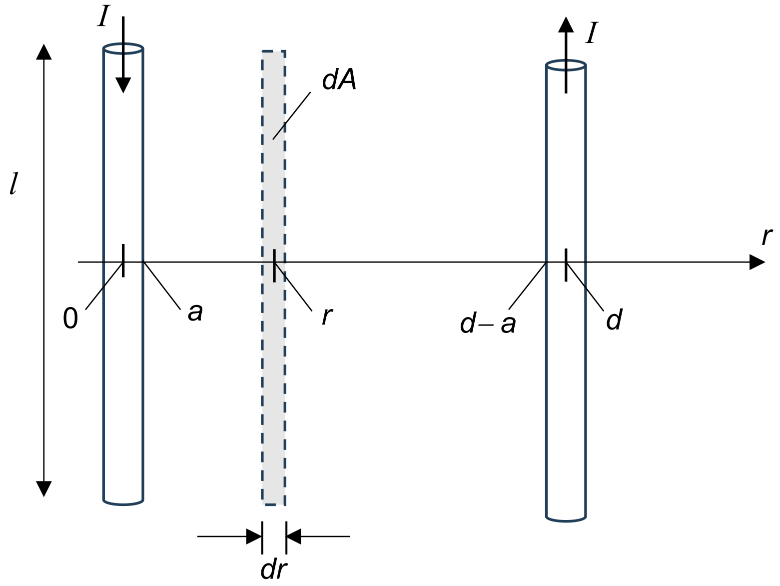
(b)

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2\left(\mu_0 i/W\right)}{i} = \frac{\pi\mu_0 R^2}{W} = \frac{\pi(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.018\text{m})^2}{0.16\text{m}} = 8.0\times10^{-9}\,\text{H}.$$

6.

Two identical long wires of radius $a = 1.53$ mm are parallel and carry identical currents in opposite directions. Their center-to-center separation is $d = 14.2$ cm. Neglect the flux within the wires but consider the flux in the region between the wires. What is the inductance per unit length of the wires?

Q6:



Consider only the region between the 2 wires of length L, the magnetic flux Φ

$$\Phi = \int_a^{d-a} B_{net}(l \times dr) = \int_a^{d-a} \left(\frac{\mu_o I}{2\pi r} + \frac{\mu_o I}{2\pi(d-r)} \right)(l \times dr)$$

$$\begin{aligned}\frac{\Phi}{l} &= \int_a^{d-a} \left(\frac{\mu_o I}{2\pi r} + \frac{\mu_o I}{2\pi(d-r)} \right) dr \\ &= \frac{\mu_o I}{2\pi} \left[\ln\left(\frac{r}{d-r} \right) \right]_a^{d-a} \\ &= \frac{\mu_o I}{\pi} \ln\left(\frac{d-a}{a} \right)\end{aligned}$$

Thus the inductance per unit length is

$$\begin{aligned}\frac{L}{l} &= \left(\frac{d\Phi}{dt} \Big/ \frac{dI}{dt} \right) \Big/ l = \frac{d\left(\frac{\Phi}{l}\right)}{dI} \\ &= \frac{\mu_o}{\pi} \ln\left(\frac{d-a}{a} \right) \\ &= \frac{4\pi \times 10^{-7}}{\pi} \ln\left(\frac{142 - 1.53}{1.53} \right) = 1.81 \times 10^{-6} \text{ H m}^{-1}\end{aligned}$$

7a

Inductors in series. Two inductors L_1 and L_2 are connected in series and are separated by a large distance so that the magnetic field of one cannot affect the other. Show that the equivalent inductance is given by

$$L_{eq} = L_1 + L_2.$$

7b

Inductors in parallel. Two inductors L_1 and L_2 are connected in parallel and separated by a large distance so that the magnetic field of one cannot affect the other. Show that the equivalent inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

Q7a:

Voltage is proportional to inductance, is proportional to resistance.
Since the (independent) voltages for series elements add ($V_1 + V_2$),
then inductances in series must add, $L_{eq} = L_1 + L_2$, just as was the case for resistances.

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (mutual inductance) The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

7b:

Voltage is proportional to inductance is proportional to resistance.
The (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$).

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt}.$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

8.

(a) The current in an RL circuit builds up to one-third of its steady-state value in 5.00 s. Find the inductive time constant.

(b) The current in an RL circuit drops from 1.0 A to 10 mA in the first second following removal of the battery from the circuit. If L is 10 H, find the resistance R in the circuit.

(c) A solenoid having an inductance of 6.30 μH is connected in series with a 1.20 kΩ resistor. (i) If a 14.0 V battery is connected across the pair, how long will it take for the current through the resistor to reach 80.0% of its final value? (ii) What is the current through the resistor at time $t = 1.0\tau_L$?

Q8a:

The steady state value of the current is also its maximum value, ε/R, which we denote as i_m. We are told that $i = i_m/3$ at $t_0 = 5.00$ s. $i = i_m\left(1 - e^{-t_0/\tau_L}\right)$, which leads to

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_m)} = -\frac{5.00\,\text{s}}{\ln(1 - 1/3)} = 12.3\text{ s.}$$

Q8b:

The current in the circuit is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant (L/R). We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L}.$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0\,\text{s}}{\ln\left((10\times10^{-3}\,\text{A})/(1.0\,\text{A})\right)} = 0.217\,\text{s.}$$

Therefore, $R = L/\tau_L = 10\text{ H}/0.217\text{ s} = 46\ \Omega$.

Q8c:

THINK The inductor in the RL circuit initially acts to oppose changes in current through it.

EXPRESS If the battery is switched into the circuit at t = 0, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$.

(a) We want to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \;\Rightarrow\; e^{-t/\tau_L} = 0.200.$$

Taking the natural logarithm of both sides, we obtain

$$-(t/\tau_L) = \ln(0.200) = -1.609.$$

Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\text{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\text{s}.$$

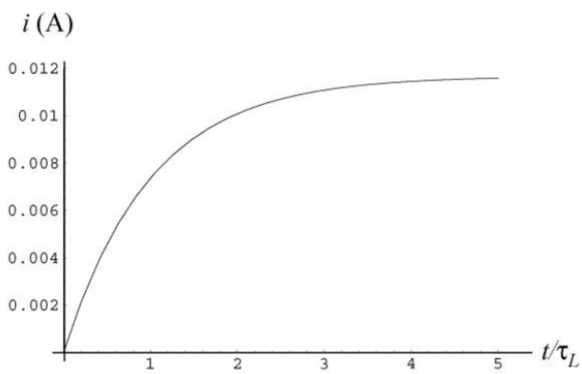
(b) At $t = 1.0\,\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20\times10^{3}\Omega}\right)(1 - e^{-1.0}) = 7.37\times10^{-3}\ \text{A}.$$

LEARN At t = 0, the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is

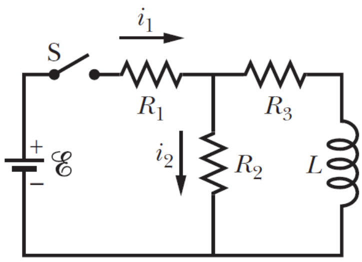
$$i_0 = \frac{\varepsilon}{R} = \frac{14.0\,\text{V}}{1.20\times10^{3}\Omega} = 0.0117\ \text{A}.$$

The current as a function of t/τ_L is plotted to the right.



9.

In Fig, $\mathscr{E}$ = 100 V, R_1 = 10.0 Ω, R_2 = 20.0 Ω, R_3 = 30.0 Ω, and L = 2.00 H. Immediately after switch S is closed, what are (a) i_1 and (b) i_2? (Let currents in the indicated directions have positive values and currents in the opposite directions have negative values.) A long time later, what are (c) i_1 and (d) i_2? The switch is then reopened. Just then, what are (e) i_1 and (f) i_2? A long time later, what are (g) i_1 and (h) i_2?



Q9:

(a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{R_1 + R_2} = \frac{100\,\text{V}}{10.0\,\Omega + 20.0\,\Omega} = 3.33\,\text{A}.$$

(b) $i_2 = i_1 = 3.33\,\text{A}$.

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0$$

$$\varepsilon - i_1 R_1 - (i_1 - i_2) R_3 = 0.$$

We solve these simultaneously for i_1 and i_2, and find

$$i_1 = \frac{\varepsilon(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{(100\,\text{V})(20.0\,\Omega + 30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 4.55\,\text{A},$$

(d) and

$$i_2 = \frac{\varepsilon R_3}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{(100\,\text{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 2.73\,\text{A}.$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 4.55 A – 2.73 A = 1.82 A. The current in R_2 is the same but in the opposite direction as that in R_3, that is, $i_2 = -1.82$ A.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.