H3 Physics Topic 3 – Special Relativity

1 An enemy spaceship moves past the earth with a speed of 0.80*c*. The captain orders the spaceship weapons department to blast the earth with pulsed laser photons every 10 seconds. For the observers on earth who see the flashes, what is the time interval they measure between photon pulses?

Ans: 16.7 s

2 At what speed does a clock have to move in order to run at a rate which is one half that of a clock at rest?

Ans: 0.866*c*

3 The distance between the Earth and Alpha Centauri is 4.2 light years (1 light year is the distance traveled by light in one year). If astronauts could travel at v = 0.95*c*, then we on Earth would assume that the trip would take the astronauts 4.2 / 0.95 = 4.4 years. The astronauts however, disagree.

a) How much time passes on the astronauts' clock?
b) What distance to Alpha Centauri do the astronauts' measure?

Ans: **(a)** 1.4 years **(b)** 1.3 lightyears.

4 An astronaut wishes to visit the Andromeda galaxy 2 million light years from Earth. He wishes the one-way trip to take him 30 years (i.e. in the frame of reference of the spaceship). Assuming that his speed is constant, how fast must he travel? *(Mathematical hint: use Binomial expansion and approximate to 1st order!)*

Ans: $(1 – 1.13 \times 10^{-10})c$

5 An alien spacecraft is flying overhead at a great distance as you stand in your backyard. You see its searchlight blink on for 0.190 s. An officer on the spacecraft measures the searchlight is on for 12.0 ms.

(a) Which of the 2 measured times is the proper time?
(b) What is the speed of the spacecraft relative to the earth?

Ans: **(a)** 12.0 ms **(b)** 0.998*c*

6 An enemy Klingon spaceship moves away from Earth at a speed of 0.80*c*. The Starship Enterprise gives chase and pursues at a speed of 0.90*c* relative to the Earth. Observers on Earth see the Starship Enterprise overtaking the enemy ship at a relative speed of 0.10*c*.

(a) With what speed is the Enterprise overtaking the Klingon, as observed by the crew of the Enterprise?
(b) Enterprise fires an electron torpedo (0.80*c* w.r.t. Enterprise). What is the speed of the torpedo w.r.t. Earth's frame?

Ans: **(a)** 0.36*c* **(b)** 0.9884*c*

7 Frame S' moves with speed $v = 0.6c$ relative to S along the $+x$-direction. Clocks in the two frames are adjusted so that $t = t' = 0$ at $x = x' = 0$.

(a) An event occurs in S at $t = 2 \times 10^{-7}$ s at $x = 50$ m. At what time does the event occur in S'?

(b) If another event occurs at $x = 10$ m and $t = 3 \times 10^{-7}$ s in S, what is the time interval between the events as measured S'?

Ans: **(a)** $t_1' = 1.25\times10^{-7}$ s, **(b)** $t_2' = 3.5\times10^{-7}$ s

8 Find the speed of an electron when it is accelerated to a kinetic energy of 1 MeV using classical mechanics. Comment on the answer and repeat the process using relativistic mechanics. (Rest mass of electron = 9.11×10^{-31} kg.)

Ans: $v = 5.93\times10^{8}$ m s^{-1}, $v = 2.82 \times 10^{8}$ m s^{-1}

9 Find the speed of a particle whose total energy is 3 times its rest energy.

Ans: $0.943c$

10 A photon torpedo contains 10^{33} photons, each having a wavelength of 200 nm. What is the momentum of the torpedo?

Ans: 3.31×10^{6} kg m s^{-1}

11 A particle of rest mass m_o and kinetic energy $2m_oc^2$ strikes and sticks to a stationary particle of rest mass $2m_o$. Find the rest mass M_o of the composite particle.

Ans: $M_o = \sqrt{17}m_o$

Suggested Solutions

1 The observer on Earth will see the photons flash at a longer interval, i.e. $\Delta t = \gamma \Delta t_p$

$$\Delta t = \gamma \Delta t_p = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta t_p = \frac{1}{\sqrt{1-\frac{(0.80c)^2}{c^2}}} \times 10 = 16.7 \text{ s}$$

2 Time dilation formula, $\Delta t = \gamma \Delta t_p$
Given that $\Delta t = 2\Delta t_p$, Lorentz factor $\gamma = 2$

$$2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$v = \sqrt{\frac{3}{4}}c = 0.866c$$

3 (a) $\gamma = 1/\sqrt{1-\frac{(0.95c)^2}{c^2}} = 3.2$

Time measured on astronauts' clock = $4.4/\gamma$ = $\frac{4.4}{3.2} = 1.4$ years

(b) Astronauts will experience length contraction by factor of γ:

$L = L_p / \gamma = \frac{4.2}{3.2}$ = 1.3 lightyears.

4 Proper length $L_p = 2 \times 10^6$ light years
Let his speed be v, Lorentz factor γ

Proper time t_p = 30 years, hence from the Earth observer frame, the trip took (30γ) years.
Since time = distance / speed,

$$t = 30\gamma = \frac{2x10^6 c}{v}$$

$$\frac{30}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2x10^6 c}{v} = \frac{2x10^6}{\frac{v}{c}}$$

Solving for $\frac{v}{c}$,

$$\frac{v}{c} = \left(1 + 2.25x10^{-10}\right)^{-\frac{1}{2}}$$

We make use of Binomial expansion, if $(1 + x)^{-1/2}$ and x is small, then we can approximate to 1st order.

i.e. $(1+x)^{-\frac{1}{2}} \approx \left(1-\frac{1}{2}x\right)$

In this case, $\frac{v}{c} \approx \left(1-\frac{1}{2}(2.25x10^{-10})\right) = 1-1.13x10^{-10}$

$v = (1 - 1.13 \times 10^{-10})\,c$

5 (a) Proper time = 12.0 ms by officer on spaceship.
(b) Using time dilation formula, $\Delta t = \gamma \Delta t_p$ and so 0.190 = 12.0 x $10^{-3}\gamma$
Solving, v = 0.998c

6 (a) $u_x' = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}} = \dfrac{0.90c - 0.80c}{1 - (0.90)(0.80)} = 0.36c$

(b) $0.80c = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}} = \dfrac{u_x - 0.90c}{1 - \dfrac{u_x (0.90c)}{c^2}}$

$u_x = 0.988c$

7 Use the Lorentz transformation equation, $t' = \gamma(t - \dfrac{v}{c^2}x)$ and substitute the necessary values. You should get the Lorentz factor = 1.25

8

1. $\dfrac{1}{2}mv^2 = 1.0 \times 10^6 \times 1.6 \times 10^{-19}$

$\Rightarrow v = \sqrt{\dfrac{2(1.0 \times 10^6 \times 1.6 \times 10^{-19})}{9.11 \times 10^{-31}}} = 5.93 \times 10^8 \text{ m s}^{-1}$

$KE = \gamma m_o c^2 - m_o c^2$

2. $1.0 \times 10^6 \times 1.6 \times 10^{-19} = \left(\dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} - 1 \right) (9.11 \times 10^{-31})(3.0 \times 10^8)^2$

$v = 2.82 \times 10^8 \text{ m s}^{-1}$

9 $E = \dfrac{1}{2}\gamma m_o c^2$

$\gamma = 3$

$\dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = 3$

$\Rightarrow v = 0.943c = 2.83 \times 10^8 \text{ ms}^{-1}$

10 $E^2 = p^2 c^2 + (m_o c^2)^2$

$\Rightarrow E = pc$

$\Rightarrow nhf = pc$

$\Rightarrow (1 \times 10^{33})(6.63 \times 10^{-34}) \left(\dfrac{\cancel{c}}{200 \times 10^{-9}} \right) = p\cancel{c}$

$p = 3.32 \times 10^6 \text{ kg m s}^{-1}$

11 momentum of the moving particle:

$$E^2 = p^2 c^2 + \left(m_o c^2\right)^2$$

$$\left[3m_o c^2\right]^2 = p^2 c^2 + \left(m_o c^2\right)^2$$

$$p^2 c^2 = 8\left(m_o c^2\right)^2$$

By principle of conservation of momentum, the final momentum of the composite particle must be the same as the momentum of the moving particle. Energy should also be conserved. Hence,

$$E^2 = p^2 c^2 + \left(m_o c^2\right)^2$$

$$\left[3m_o c^2 + 2m_o c^2\right]^2 = 8\left(m_o c^2\right)^2 + \left(M_o c^2\right)^2$$

$$25{m_o}^2 = 8{m_o}^2 + {M_o}^2$$

$$M_o = \sqrt{17}m_o$$