<u>H3 Superposition Tutorial</u>

1 **(a)** Fig. 1.1 shows the equilibrium and actual positions at an instant in time of a series of particles forming part of a stationary sound wave. Particle M is at its maximum displacement.

(i) On Fig. 1.2, sketch the variation of the displacement with distance, taking right as positive. Label your sketch with "P". [2]

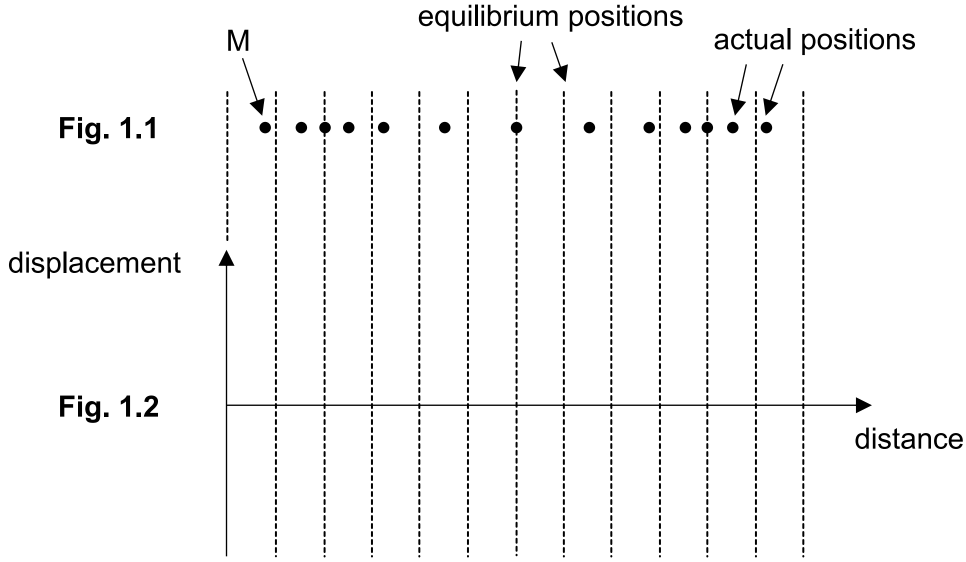


Fig. 1.1

Fig. 1.2

(ii) On Fig. 1.2, sketch the variation of displacement with distance $\frac{1}{4}$ period later. Label your sketch with "Q". [1]

(b) The standing wave in **(a)** is formed in a pipe that is closed at one end and open at the other. Sound waves from a coherent source travel parallel to the axis of the pipe. They reflect off the wall of the pipe, subsequently meet and interfere with sound waves from the same source, as shown in Fig. 1.3.

Note that there is a π-phase change upon reflection off the wall.

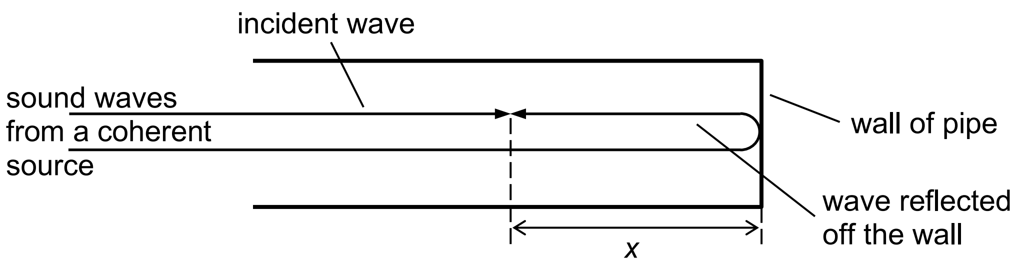


Fig. 1.3

(i) State the condition for the formation of a displacement antinode.

(ii) Hence, determine the shortest distance x (as a multiple of the wavelength, λ) from the wall of the pipe at which a displacement antinode is formed.

2 **(a)** Fig. 2.1 shows two point-sources of waves, S_1 and S_2. P is a point equidistant from the two sources.

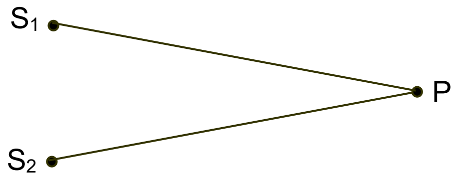


Fig. 2.1

The sources radiate waves of the same wavelength. The amplitudes at P due to S_1 and S_2 are A_1 and A_2, respectively. At P, the time variation of the disturbance due to the wave from S_1 may be represented by $y_1 = A_1 \cos \omega t$.

(i) If, at P, there is a phase angle ϕ between the waves from the two sources, write down an expression for the disturbance y_2 due to the wave from S_2.

(ii) By a graphical method, or otherwise, show that A_r, the amplitude of the resultant displacement at P, is given by

$$A_r^{\ 2} = A_1^{\ 2} + A_2^{\ 2} + 2A_1A_2 \cos \phi$$ [4]

(b) Fig. 2.2 shows two vertical slits Q_1 and Q_2 in an opaque screen, a distance *d* apart.

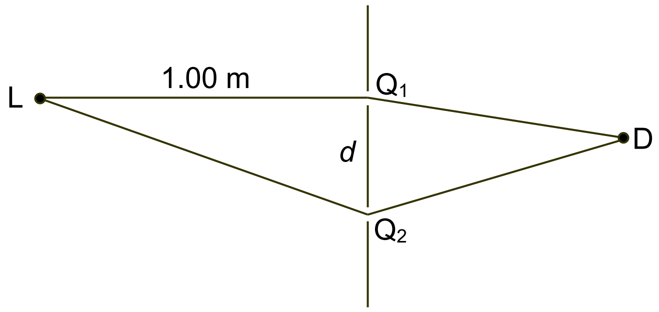


Fig. 2.2

A vertical strip-light source producing monochromatic light of wavelength 590 nm is set up at L, exactly one metre from Q_1 on the normal to the screen, so that both slits are illuminated. A detector is placed at D, exactly equidistant from the slits on the other side of the screen. Readings of the light intensity at D are taken as follows: I_1 with the slit Q_2 covered but Q_1 open; I_2 with Q_1 covered but Q_2 open; and I_r with both slits open. It is found that I_r is exactly equal to the sum of I_1 and I_2.

(i) By reference to the relationship given in **(a)(ii)** above, deduce the separation *d* of the slits. [7]

(ii) Is this the only possible value for *d*? Explain. [2]

[N92/Q4]

3 In a double-slit experiment for the measurement of the wavelength of light from a laser, the two slits are 1.00 mm apart, and are 3.00 m in front of a screen placed parallel to the plane of the slits.

(a) The distance of the fifth bright fringe from the central bright fringe is found to be 9.50 mm. what is the wavelength of the laser light? [4]

The experimenter finds that the measurements of the separation of the bright fringes from the central bright fringe are subject to an uncertainty *u* which depends on the order *n* of the fringe (the number of the fringes away from the central bright fringe). This uncertainty is given by

$$u = 0.010 + 0.0004n^2$$

where *u* is measured in mm, and n takes the values of 1, 2, 3 … according to the order of the fringe.

(b) Suggest two practical reasons why the uncertainty might be expected to be larger for larger values of *n*. [2]

(c) Because of the uncertainty *u* in the fringe separation, the value of the wavelength you calculated in **(i)** is also subject to an uncertainty. Find this uncertainty. Express the wavelength, with the associated uncertainty, to an appropriate number of significant figures. [4]

(d) To obtain the least uncertainty in the values of the wavelength, on what order of fringe should the measurements be taken? [4]

[N93/Q5(c)]

4 **(a)** In Fig. 4.1, S_1 and S_2 are two equal point sources a distance *d* apart.

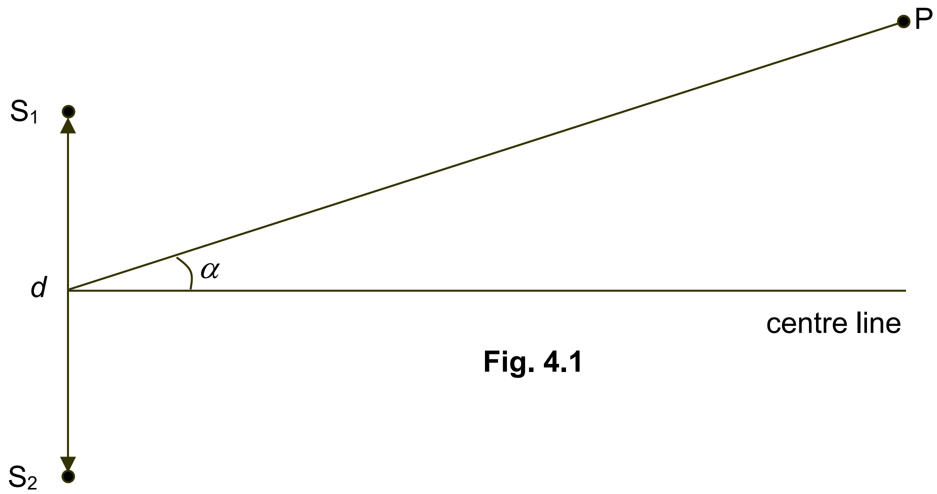


Fig. 4.1

The sources emit waves of wavelength λ in phase. P is a point on the line passing through the midpoint of S_1S_2 and making an angle α with the centre line. The distances of P from S_1 and S_2 are r_1 and r_2 respectively. The equation for the displacement y_1 of the wave arriving at P from S_1 is

$$y_1 = y_o \sin\left[\left(\frac{2\pi}{\lambda}\right)r_1 - \omega t\right] \qquad \text{equation 4.1}$$

(i) Write down an equation, similar to equation 4.1, for the displacement y_2 of the waves arriving at P from S_2. [1]

(ii) Hence, or otherwise, write down an expression for the phase difference ϕ between the waves arriving at P from S_1 and S_2. [1]

(iii) When P is a very great disatnce from the sources, S_1P is approximately parallel to S_2P. Making this approximation, find an expression for ϕ in terms of α, d and λ. [2]

(iv) Hence, obtain the condition for P to be a point at which the amplitude of the resultant disturbance is a maximum. [3]

(b) A stereo system in a large hall has two identical speakers, S_1 and S_2, 1.2 m apart. The amplitude of the output of each speaker is proportional to the voltage across its terminals. The voltage input to each speaker is adjusted by means of a balance control. The arrangement is illustrated in Fig. 4.2.

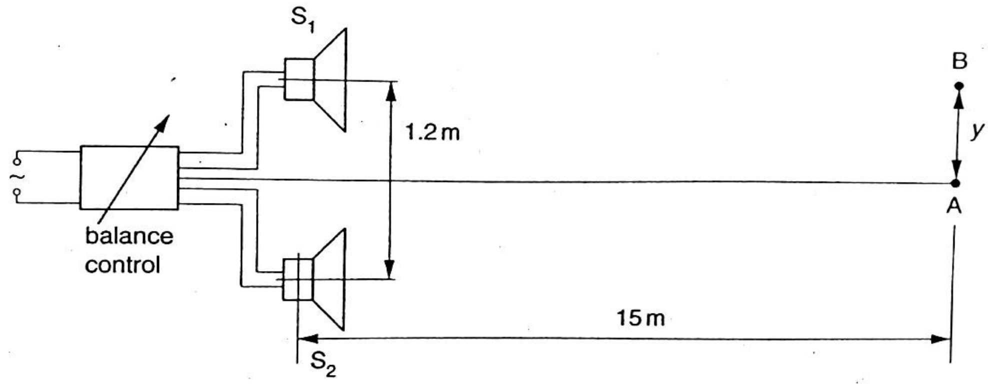


Fig. 4.2

Initilally, the speakers are emitting signals of frequency 1000 Hz which are in phase. The balance control is set so that there is a voltage of 6.0 V r.m.s. across each speaker. An observer stands on the centre line at the point A, 15 m away from the point half-way between the speakers, and hears a loud sound of intensity I_{max}. As he moves along the line at right angles ot the centre line, he observes that the intensity of the sound first falls to zero at the point B, a distance y from A. The speed of sound in the air in the hall is 330 m s^{-1}.

(i) Estimate the distance y. State any assumptions you make. [4]

(ii) At what other frequencies of operation of the speakers would the point B also be a position of zero intensity? [2]

(iii) With the speakers emitting the original signal of frequency 1000 Hz, the balance control is now adjusted so that the voltages across S_1 and S_2 are 3.0 V r.m.s. and 9.0 V r.m.s. respectively.

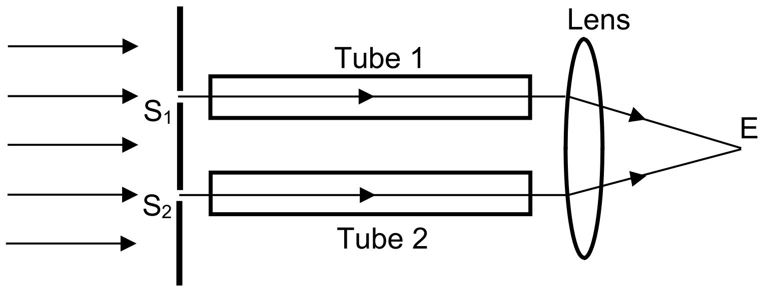
1. In terms of I_{max}, find the new intensities at points A and B.
2. Show that, although the average of the voltages across the speakers is the same as before (i.e. 6.0 V r.m.s.), the adjustment of the balance control has caused a change in the total power output of the system. Discuss this result with reference to the principle of conservation of energy. [5]

[N98/Q9]

5 The mercury spectrum contains two intense yellow lines with wavelengths 577.0 nm and 579.1 nm respectively. Light from a mercury lamp is incident normally on a diffraction grating ruled with 4300 lines per centimetre. To observe the yellow lines clearly, it is desirable that there should be an angular separation of at least 0.20° between the diffracted beams produced by light of the two wavelengths.
Which orders of diffraction could be used? Suggest, with a reason, which one would be used.

[4]
[N2001/3]

6 Parallel coherent light of wavelength 519 nm in vacuum is incident upon two slits S_1 and S_2. The light emerging from the slits passes through two identical evacuated tubes and is then superposed and viewed in the region around E.



(a) When some gas is allowed to leak gradually into Tube 2, the intensity of the light at point E is found to change periodically. Explain how this observation leads to the conclusion that the speed of light in the gas is different from that in a vacuum.
You may assume that the frequency of the light is constant.

(b) The light is observed to undergo 312 complete cycles in the variation of its intensity at E as the gas is introduced into the tube.

Given that the length of each tube is 62.0 cm, obtain a numerical value for the ratio

$$\frac{\text{speed of light in vacuum}}{\text{speed of light in the gas}}$$

7 The objective lens of a telescope has a diameter of 12.0 cm. At what distance would two small green objects 30.0 cm apart be just resolved by the telescope, assuming the resolution is limited only by diffraction by the objective lens?

Assume that the wavelength is 540 nm.

8 Light is incident normally on a double-slit. The width of each slit is 0.140 mm while the spacing between the centre of each slit is 0.840 mm. An interference pattern is observed on a screen which is placed a distance away from the double-slit.

It is observed that some orders of the diffraction maxima are missing from the interference pattern on the screen.

Determine the missing orders.

9 This question is about wave interference.

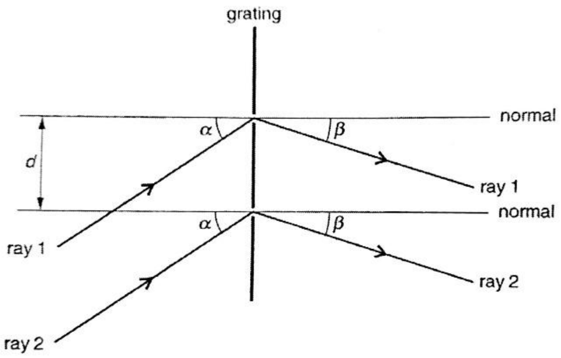
(a) The term phase difference and path difference are used in describing the superposition of waves and interference phenomena.

(i) Distinguish between phase difference and path difference. [2]

(ii) State the mathematical relationship linking the phase difference $\Delta\Phi$ and path difference Δs between two waves. [1]

(iii) State the mathematical relationship involving phase difference which must be satisfied if two waves traveling in the same direction are to interfere constructively. [1]

(b) A parallel beam of coherent, monochromatic light of wavelength λ is incident on a diffraction grating of spacing *d* at an angle α to the normal, as shown in the figure below.



On passing through the grating, the light is strongly diffracted at certain values of the angle β to the normal. Each of these values defines a different order of diffraction.

(i) **1.** Write down an expression for the path difference between ray 1 and ray 2 as the rays, incident at angle α, arrive at the grating. [1]

2. Hence obtain an expression for the total path difference between ray 1 and ray 2 for rays incident at angle α, passing through the grating, and diffracted at angle β, as shown in the figure. [2]

3. Use the expression to show that the condition for strong (i.e. constructive) diffraction is

$$d(\sin\alpha + \sin\beta) = m\lambda$$

where *m* is an integer. [1]

(ii) You derived the equations in **(i)** by considering two slits only. Real diffraction gratings have very many slits, not just two. State two practical disadvantages of using a two-slit grating to measure wavelength. [2]

(c) A parallel beam of coherent, monochromatic light is incident on a diffraction rating at an angle α to the normal. First-order diffraction is observed in the transmitted light at an angle 12.5° to the normal on one side of the normal and 49.7° to the normal on the other side of the normal.

(i) Draw a labelled diagram showing the grating, the direction of the incident light and the two first-order diffractions. [1]

(ii) Calculate the angle α. [3]

(iii) The wavelength of the light used is 633 nm. Determine the grating spacing. [1]

(iv) Identify any higher order diffraction directions. Draw and label any such directions in your diagram in **(i)**. Show your working. [5]

[N2005/8]

10 This question is on the Michelson Interferometer.

(a) How far must the moveable mirror of a Michelson interferometer be displaced for 2500 fringes of the red cadmium line ($\lambda = 643.8\ nm$) to cross the centre of the field of view? [3]

(b) Find the angle subtended by the radius of the tenth bright fringe with respect to the centre of the field of view in a Michelson interferometer when the central-path difference, *2d*, is 1.50 mm.

Assume the orange light of a krypton arc ($\lambda = 605.8\ nm$) is used and that the interferometer is adjusted in each case so that the first bright fringe forms a maximum at the centre of the pattern. [4]

<u>Numerical Answers</u>

1 **(b)** **(ii)** $\lambda/4$

2 **(b)** **(i)** 0.543 mm

3 **(a)** 633 nm

(c) $(6.33 \pm 0.01) \times 10^{-7}$ m

(d) fifth order

4 **(b)** **(i)** 2.1 m

(ii) 3000 Hz, 5000 Hz, 7000 Hz …

5 Third and fourth orders, but third is preferred.

6 **(b)** 1.00026

7 54.6 km

8 6, 12 ,18, 24 …

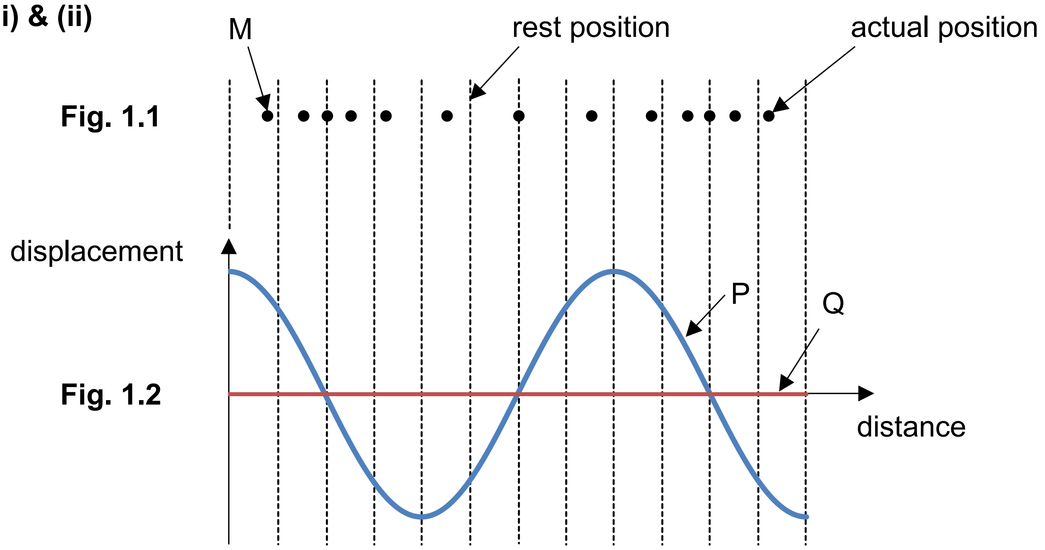
9 **(c)** **(ii)** 15.8°

(iii) 1.29×10^{-6} m **(iv)** max order below normal = 2, at 45.2°

10 **(a)** 8.048×10^{-4} m **(b)** 4.887°

Suggested Solutions to H3 Tutorial on Topics (12) Superposition

1 (a) (i) & (ii)



(b) (i) To form a displacement antinode, the two waves must meet in-phase (or with phase difference $= 2n\pi$, where n is an integer). B1

(ii) The path difference between the incident and reflected waves is $2x$. C1

At an antinode, the phase difference must be integer multiple of 2π:

phase difference $= \dfrac{2x}{\lambda} \times 2\pi + \pi = 2n\pi$ where n is an integer M1

$x = (2n-1)\pi \times \dfrac{\lambda}{4\pi} = \dfrac{2n-1}{4}\lambda$

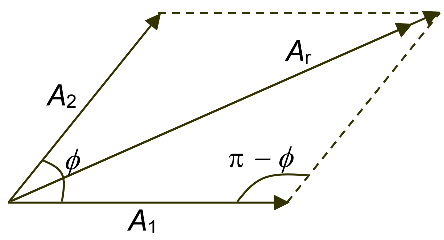
Hence, the shortest distance is

$x = \dfrac{\lambda}{4}$, A1

corresponding to $n = 1$.

2 (a) (i) $y_2 = A_2 \cos(\omega t + \phi)$

(ii) Phasor diagram:



$A_r^2 = A_1^2 + A_2^2 - 2A_1A_2\cos(\pi - \phi)$

$A_r^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi)$ (shown)

Alternative method 1:

$$y_r = y_1 + y_2$$

$$A_r \cos(\omega t + \delta) = A_1 \cos(\omega t) + A_2 \cos(\omega t + \phi) \qquad \text{... (1)}$$

Substitute t = 0 into (1),

$$A_r \cos(\delta) = A_1 + A_2 \cos(\phi)$$

$$\cos\delta = \frac{A_1 + A_2 \cos\phi}{A_r} \qquad \text{...(2)}$$

Substitute $\omega t = \frac{\pi}{2}$ into (1),

$$A_r \cos\left(\frac{\pi}{2} + \delta\right) = A_2 \cos\left(\frac{\pi}{2} + \phi\right)$$

$$A_r \sin\delta = A_2 \sin\phi$$

$$\sin\delta = \frac{A_2}{A_r}\sin\phi \qquad \text{... (3)}$$

Since $\sin^2\delta + \cos^2\delta = 1$

$$\left(\frac{A_2}{A_r}\sin\phi\right)^2 + \left(\frac{A_1 + A_2\cos\phi}{A_r}\right) = 1$$

$$A_2^{\ 2}\sin^2\phi + (A_1 + A_2\cos\phi)^2 = A_r^{\ 2}$$

$$A_r^{\ 2} = A_1^{\ 2} + A_2^{\ 2} + 2A_1A_2\cos\phi \qquad \text{(shown)}$$

Alternative method 2:

$$y_r = y_1 + y_2$$

$$A_r \cos(\omega t + \delta) = A_1 \cos(\omega t) + A_2 \cos(\omega t + \phi)$$

Using $\omega t = \frac{\omega t + (\omega t + \phi)}{2} + \frac{\omega t - (\omega t + \phi)}{2}$ and

$(\omega t + \phi) = \frac{\omega t + (\omega t + \phi)}{2} - \frac{\omega t - (\omega t + \phi)}{2}$,

$$A_1 \cos(\omega t) = A_1 \cos\left(\frac{\omega t + (\omega t + \phi)}{2} + \frac{\omega t - (\omega t + \phi)}{2}\right)$$

$$= A_1\left[\begin{array}{l}\cos\left(\frac{\omega t + (\omega t + \phi)}{2}\right)\cos\left(\frac{\omega t - (\omega t + \phi)}{2}\right) \\ -\sin\left(\frac{\omega t + (\omega t + \phi)}{2}\right)\sin\left(\frac{\omega t - (\omega t + \phi)}{2}\right)\end{array}\right]$$

$$= A_1\left[\cos\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) - \sin\left(\omega t + \frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)\right]$$

Similarly,

$$A_2\cos(\omega t+\phi)=A_2\cos\left(\frac{\omega t+(\omega t+\phi)}{2}-\frac{\omega t-(\omega t+\phi)}{2}\right)$$

$$=A_2\begin{bmatrix}\cos\left(\frac{\omega t+(\omega t+\phi)}{2}\right)\cos\left(\frac{\omega t-(\omega t+\phi)}{2}\right)\\+\sin\left(\frac{\omega t+(\omega t+\phi)}{2}\right)\sin\left(\frac{\omega t-(\omega t+\phi)}{2}\right)\end{bmatrix}$$

$$=A_2\left[\cos\left(\omega t+\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)+\sin\left(\omega t+\frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)\right]$$

Hence,

$$A_r\cos(\omega t+\delta)=A_1\cos(\omega t)+A_2\cos(\omega t+\phi)$$

$$=A_1\left[\cos\left(\omega t+\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)-\sin\left(\omega t+\frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)\right]$$

$$+A_2\left[\cos\left(\omega t+\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)+\sin\left(\omega t+\frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)\right]$$

$$=\left[(A_1+A_2)\cos\left(\frac{\phi}{2}\right)\right]\cos\left(\omega t+\frac{\phi}{2}\right)-\left[(A_1-A_2)\sin\left(\frac{\phi}{2}\right)\right]\sin\left(\omega t+\frac{\phi}{2}\right)$$

The amplitude A_r is thus

$$A_r=\sqrt{\left[(A_1+A_2)\cos\left(\frac{\phi}{2}\right)\right]^2+\left[(A_1-A_2)\sin\left(\frac{\phi}{2}\right)\right]^2}$$

$$=\sqrt{A_1^2+A_2^2+2A_1A_2\left(\cos^2\left(\frac{\phi}{2}\right)-\sin^2\left(\frac{\phi}{2}\right)\right)}$$

$$=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}\qquad\text{(shown)}$$

(b) **(i)** Since $\text{Intensity}\propto(\text{Amplitude})^2$,

$I_r=I_1+I_2\Rightarrow A_r^2=A_1^2+A_2^2$

Comparing with the expression in **(a)(ii)**,

$$2A_1A_2\cos\phi=0\Rightarrow\phi=\frac{\pi}{2}$$

The light from Q_1 and Q_2 meet at D with a phase difference of $\frac{\pi}{2}$. This corresponds to a path difference of $\frac{\lambda}{4}$.

$$\text{path difference}=LQ_2-LQ_1=\frac{\lambda}{4}$$

$$LQ_2-1.00=\frac{\lambda}{4}$$

$$\sqrt{1+d^2}-1.00=\frac{590\times10^{-9}}{4}$$

$$d=5.43\times10^{-4}\,\text{m}$$

(ii) This is not the only possible value of *d*.

ϕ can be any odd integer multiple of $\frac{\pi}{2}$ (i.e. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots, \frac{(2n+1)\pi}{2}$ where *n* is an integer).

3 **(a)** Using $x = \frac{\lambda D}{a}$

$$9.50 \times 10^{-3} = 5x = 5\frac{\lambda D}{a}$$

$$\lambda = \frac{(9.50 \times 10^{-3})(1.00 \times 10^{-3})}{5(3.00)}$$

$$= 6.33 \times 10^{-7} \text{ m}$$

(b)
1. Due to single slit diffraction effects, the intensity of the bright fringes decrease with distance from the central bright fringe. Hence, the higher order fringes would be dimmer (hence less well-defined borders) and thus affect the measurements of fringe separation.
2. The fringe separation increases for larger values of *n*. Hence, the uncertainty would be expected to be larger.

Let L_n be the distance of the *n*th bright fringe from the central bright fringe. $x_n = \frac{L_n}{n}$ is not a constant value (because the screen is a plane parallel to the plane of the two slits). It increases with higher orders.
Uncertainty in *x* = $(x_n - \text{constant value of } x \text{ expected})$
Hence, the uncertainty in *x* increases with higher orders.

(c) When *n* = 5, *u* = 0.010 + 0.0004 (5)^2 = 0.02 mm

$$\Delta x = \frac{u}{5} = \frac{0.02 \times 10^{-3}}{5} = 4 \times 10^{-6} \text{ m}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta x}{x}$$

$$\Delta\lambda = \frac{(4 \times 10^{-6})}{(9.5 \times 10^{-3}/5)}(6.33 \times 10^{-7})$$

$$= 1.3 \times 10^{-9} \text{ m}$$

$$\lambda \pm \Delta\lambda = (633 \pm 1) \times 10^{-9} \text{ m}$$

$x = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{\Delta x d}{D}$

$\Rightarrow \frac{\Delta\lambda}{\lambda} = 0$

(d) Uncertainty of fringe separation $\Delta x = \frac{u}{n} = \frac{0.010}{n} + 0.0004n$

For uncertainty to be minimum, $\frac{d\left(u/n\right)}{dn} = -\frac{0.010}{n^2} + 0.0004 = 0$

n = 5

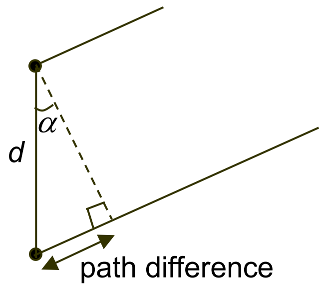
3 **(a)** **(i)**

$$y_2 = y_o \sin\left[\left(\frac{2\pi}{\lambda}\right)r_2 - \omega t\right]$$

(ii)

$$\phi = \left[\left(\frac{2\pi}{\lambda}\right)r_2 - \omega t\right] - \left[\left(\frac{2\pi}{\lambda}\right)r_1 - \omega t\right] = \left(\frac{2\pi}{\lambda}\right)(r_2 - r_1)$$

(iii)



path difference $= d\sin\alpha = r_2 - r_1$

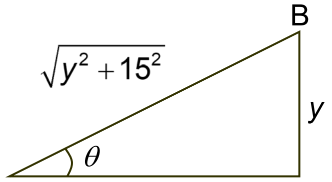
$$\phi = \left(\frac{2\pi}{\lambda}\right) d\sin\alpha$$

(iv) For P to be a maxima, $\phi = 2n\pi$

$$\therefore 2n\pi = \left(\frac{2\pi}{\lambda}\right) d\sin\alpha$$

path difference $= d\sin\alpha = n\lambda$

(b) **(i)** At point B,



$$d\sin\theta = \frac{\lambda}{2}$$

$$(1.2)\left(\frac{y}{\sqrt{y^2+15^2}}\right) = \frac{330/1000}{2}$$

$$\frac{y^2}{y^2+15^2} = 0.01891$$

$$y = 2.1\text{ m}$$

To use $d\sin\theta = \frac{\lambda}{2}$, one assumes that the distance to from the speakers to the detector is sufficiently large compared to the separation between the speakers such that the waves from the two speakers are approximately parallel.

(ii) When f = 1000 Hz,

$$d\sin\theta = \frac{\lambda}{2} = \frac{v}{2f}$$

$$f = \frac{v}{2d\sin\theta} = 1000\text{ Hz}$$

For point B to always be a point of destructive interference,

$$d\sin\theta = (2n+1)\frac{\lambda}{2} = (2n+1)\frac{v}{2f}$$

$$f = (2n+1)\frac{v}{2d\sin\theta} = (2n+1)(1000)$$ where n is an integer

Thus, other possible frequencies include 3000 Hz, 5000 Hz, 7000 Hz …

(iii) **1.** Initially, at point A, $I_{\max} \propto (A_1 + A_2)^2 = (6.0 + 6.0)^2 = 12^2$

After adjustment, at point A,

$I_A \propto (3.0 + 9.0)^2 = 12^2$

$\therefore I_A = I_{\max}$

After adjustment, at point B,

$I_B \propto (9.0 - 3.0)^2 = 6^2$

$\therefore I_B = \dfrac{1}{4} I_{\max}$

2. The waves from the two speakers interfere to form a pattern, in which the intensities vary from some I_{max} to some I_{min}. The total power delivered over the entire pattern (an integration of intensity times area over the whole screen), however, must be equal to the sum of the power delivered by each speaker alone (without any interference), so that energy is conserved.

[**Note**: Interference causes redistribution of energy.]

power emitted by each speaker alone = intensity × area of wavefront
Since the waves from the two speakers spread out over the same wavefront,
power emitted by a speaker $\propto$ intensity $\propto$ amplitude^2 $\propto$ $(V_{rms})^2$
$\Rightarrow$ power emitted by a speaker = $k(V_{rms})^2$, k being a constant.

Originally,
total power of two speakers = $k(6.0)^2 + k(6.0)^2 = 72k$
After the adjustment,
total power of two speakers = $k(3.0)^2 + k(9.0)^2 = 90k$

Hence, the total power emitted increases after the adjustment.

5 $d \sin\theta = n\lambda_1$

$d \sin(\theta + 0.2°) = n\lambda_2$

$$\frac{\sin(\theta + 0.2°)}{\sin\theta} = \frac{579.1}{577}$$

$\theta = 43.76°$

Substitute into above equation:

$$n = \frac{d \sin\theta}{\lambda_1} = \left(\frac{1\times10^{-2}}{4300}\right)\left(\frac{\sin 43.76°}{577\times10^{-9}}\right) = 2.8$$

Hence, 3rd or any higher order bright fringes could be used. But 3rd order is preferred, since the lines would be brighter/more visible.
(Intensity decreases as order increases, due to single slit diffraction. It can also be shown that the highest order is 4th).

6 **(a)** The periodically changing intensity of light at point E indicates that the light from the tubes that meet at E is undergoing constructive and destructive interference periodically. This means that the optical path difference between the light rays is changing periodically. Since the length of the tubes are the same and the frequency of the light is constant, the changing path difference must be due to the changing wavelength in Tube 2. Since .

(b) No. of cycles in Tube 1 (vacuum) = $\dfrac{\text{length of tube}}{\text{wavelength}} = \dfrac{62\times10^{-2}}{519\times10^{-9}} = 1194605.01$

No. of cycles in Tube 2 (gas) = 1194605.01 + 312 = 1194917.01

$$\frac{v_{\text{vacuum}}}{v_{\text{gas}}} = \frac{\lambda_{\text{vacuum}}}{\lambda_{\text{gas}}} = \frac{(0.62/\text{no. of cycles in vacuum})}{(0.62/\text{no. of cycles in gas})} = \frac{\text{no. of cycles in gas}}{\text{no. of cycles in vacuum}}$$
$$= \frac{1194917.01}{1194605.01} = 1.00026$$

7 For circular apertures, $\sin\theta = 1.22\dfrac{\lambda}{D} = \dfrac{x}{R}$ (using small angle approximation).

$$R = \frac{x}{1.22\lambda}D = \frac{0.300}{1.22\left(540\times10^{-9}\right)}(0.12) = 54.6\times10^{3}\text{ m}$$

8 For single slit, $a\sin\theta = m\lambda$ (where a is the slit width)
For double slit, $d\sin\theta = n\lambda$ (where d is the slit separation)
The missing orders occur when the double slit maxima coincide with the single slit minima. At these points, θ is the same.

$$\therefore \frac{d\sin\theta}{a\sin\theta} = \frac{n\lambda}{m\lambda}$$
$$\frac{n}{m} = \frac{d}{a} = \frac{0.840}{0.140} = 6$$

When m = 1 , n = 6
m = 2 , n = 12
m = 3 , n = 18
Hence, the missing orders are 6, 12, 18, 24 etc (i.e. multiples of 6)

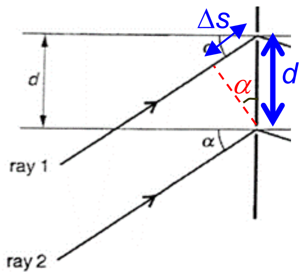
9 **(a)** **(i)** Phase difference between two waves is a measure of the fraction of a cycle which one is ahead of the other. It is expressed in either radians or degrees.

Path difference between two waves is the difference in the distances that each wave travels from its source to the point where the two waves meet. It is expressed in terms of number of wavelengths.

(ii) $\Delta\Phi = \left(\dfrac{\Delta s}{\lambda}\right)2\pi$ (in radians)

(iii) $\Delta\Phi = 2m\pi,$ where m is an integer

(b) (i) 1.



The path difference between the 2 incident rays is $d\sin\alpha$.

2. Total path difference $\Delta s = d\sin\alpha + d\sin\beta$

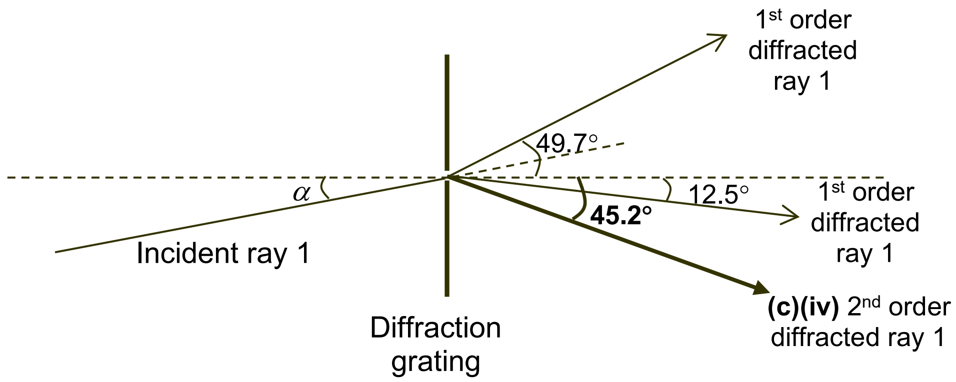
3. For constructive interference, the path difference of the two waves should be an integer-multiple of the wavelength.

$$d\sin\alpha + d\sin\beta = m\lambda$$

where m is an integer.

(ii) The brightness of the interference fringes produced by a two-slit grating is much lower than that of the fringes produced by a diffraction grating.
The interference fringes produced by a two-slit grating are not as sharp and distinguished as those produced by a diffraction grating.
Hence, the interference fringes produced by a diffraction grating are more easily detected.

(c) (i)



(ii) The 1st order diffracted rays **above** the normal would have a path difference of

$$d\left(\sin 49.7° - \sin\alpha\right) = \lambda \qquad \text{-------- (1)}$$

(path length of diffracted ray 2 exceeds that of diffracted ray 1)

The 1st order diffracted rays **below** the normal would have a path difference of

$$d\left(\sin\alpha + \sin 12.5°\right) = \lambda \qquad \text{-------- (2)}$$

(path length of diffracted ray 1 exceeds that of diffracted ray 2)

Equating the 2 expressions:

$$\left(\sin 49.7° - \sin\alpha\right) = \left(\sin\alpha + \sin 12.5°\right)$$

$$2\sin\alpha = \sin 49.7° - \sin 12.5° = 0.5462$$

$$\alpha = 15.8°$$

(iii) Using equation (1) in **(c)(ii)**,

$$d\left(\sin 49.7° - \sin 15.8°\right) = 6.33\times10^{-7}$$

$$d = 1.29\times10^{-6}\ \text{m}$$

(iv) In the direction **above the normal**, the maximum order is when the diffracted ray is 90° anticlockwise of the normal. Hence,

$d\left(\sin 90° - \sin 15.8°\right) = m\lambda$

$$m = \frac{\left(1.29 \times 10^{-6}\right)\left(\sin 90° - \sin 15.8°\right)}{6.33 \times 10^{-7}} = 1.48$$

Hence, maximum order above the normal is **1** (at 49.7°).

In the direction **below the normal,** the maximum order is when the diffracted ray is 90° aclockwise of the normal. Hence,

$d\left(\sin 15.8° + \sin 90°\right) = m\lambda$

$$m = \frac{\left(1.29 \times 10^{-6}\right)\left(\sin 15.8° + \sin 90°\right)}{6.33 \times 10^{-7}} = 2.59$$

Hence, maximum order below the normal is **2**.
To determine angle of diffracted beam for maximum order below the normal:

$d\left(\sin 15.8° + \sin \beta\right) = 2\lambda$

$$\beta = \sin^{-1}\left[\frac{2\left(6.33 \times 10^{-7}\right)}{\left(1.29 \times 10^{-6}\right)} - \sin 15.8°\right] = 45.2°$$

(refer to sketch in the earlier diagram)

10 **(a)** A ring disappears each time $2d$ decreases by λ.
Hence, when 2500 rings disappear, $2d = 2500\lambda$

$\Rightarrow\ d = 1250\lambda = 1250 \times 643.8 \times 10^{-9} = 8.048 \times 10^{-4}$ m

(b) For formation of bright rings, $2d\cos\theta = m\lambda$

At centre of ring system, $\theta = 0^0 \Rightarrow \cos\theta = 1 \rightarrow \therefore 2d = m_0\lambda$ at the centre

$$\Rightarrow m_0 = \frac{2d}{\lambda} = \frac{1.50 \times 10^{-3}}{605.8 \times 10^{-9}} = 2476.065$$

For the 10th bright fringe (or ring), $m = m_0 - 9$

$$\Rightarrow \frac{2d\cos\theta}{2d} = \frac{m\lambda}{m_0\lambda} = \frac{2467.065}{2476.065} \Rightarrow \theta = 4.887^0$$