Tutorial 2 Rotational Energy and Rotational Dynamics

Rotational Energy

1 A tall, cylindrical chimney falls over when its base is ruptured. Treat the chimney as a thin rod of length 55.0 m.

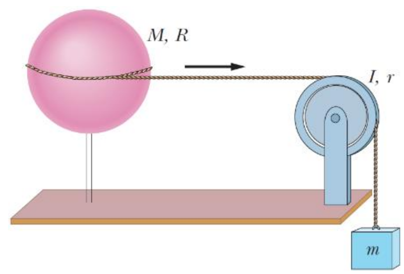
At the instant it makes an angle of 35.0° with the vertical as it falls, what are

(a) the radial acceleration of the top, and

(b) the tangential acceleration of the top. (*Hint:* Use energy considerations, not a torque.)

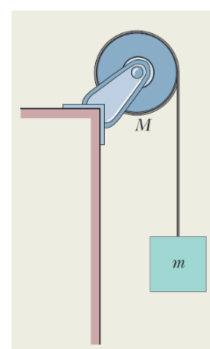
(c) At what angle θ is the tangential acceleration equal to g?

2 A uniform spherical shell of mass M = 4.5 kg and radius R = 8.5 cm can rotate about a vertical axis on frictionless bearings. A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}$ kg m^2 and radius r = 5.0 cm, and is attached to a small object of mass m = 0.60 kg. There is no friction on the pulley's axle; the cord does not slip on the pulley.



Using energy consideration, determine the speed of the object when it has fallen 82 cm after being released from rest.

3 In the figure a wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is 0.40 kg m^2 . A massless cord wrapped around the wheel's circumference is attached to a 6.0 kg box. The system is released from rest.



When the box has a kinetic energy of 6.0 J, what are

(a) the wheel's rotational kinetic energy and

(b) the distance the box has fallen?

4 A uniform cylinder of radius 10 cm and mass 20 kg is mounted so as to rotate freely about a horizontal axis that is parallel to and 5.0 cm from the central longitudinal axis of the cylinder.

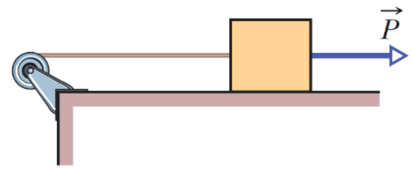
(a) What is the rotational inertia of the cylinder about the axis of rotation?

(b) If the cylinder is released from rest with its central longitudinal axis at the same height as the axis about which the cylinder rotates, what is the angular speed of the cylinder as it passes through its lowest position?

5 A wheel of radius 0.20 m is mounted on a frictionless horizontal axle. A massless cord is wrapped around the wheel and attached to a 2.0 kg box that slides on a frictionless surface inclined at angle θ = 20° with the horizontal. The box accelerates down the surface at 2.0 m s^{-2}.

What is the rotational inertia of the wheel about the axle?

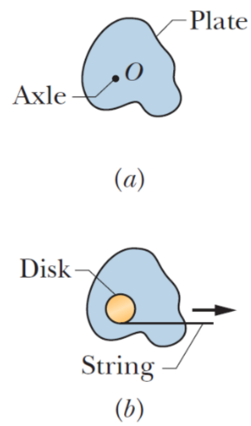
6 A wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is 0.050 kg m^2. A massless cord wrapped around the wheel is attached to a 2.0 kg block that slides on a horizontal frictionless surface. If a horizontal force of magnitude P = 3.0 N is applied to the block as shown in the figure, what is the magnitude of the angular acceleration of the wheel?



State any assumptions that you made in your calculation.

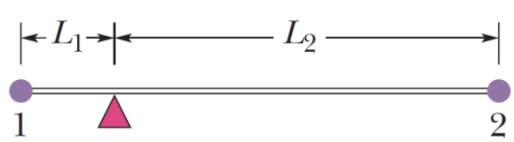
7 In Fig. (a), an irregularly shaped plastic plate with uniform thickness and density (mass per unit volume) is to be rotated around an axle that is perpendicular to the plate face and through point *O*.

The rotational inertia of the plate about that axle is measured with the following method. A circular disk of mass 0.500 kg and radius 2.00 cm is glued to the plate, with its center aligned with point *O* Fig. (b). A string is wrapped around the edge of the disk the way a string is wrapped around a top. Then the string is pulled for 5.00 s. As a result, the disk and plate are rotated by a constant force of 0.400 N that is applied by the string tangentially to the edge of the disk. The resulting angular speed is 114 rad/s.



What is the rotational inertia of the plate about the axle?

8 The figure shows particles 1 and 2, each of mass m, fixed to the ends of a rigid massless rod of length $L_1 + L_2$, with L_1 = 20 cm and L_2 = 80 cm. The rod is held horizontally on the fulcrum and then released.



What are the magnitudes of the initial accelerations of

(a) Particle 1, and

(b) Particle 2?

Answers

1		
	(a)	5.32 m s^{-2}
	(b)	8.44 m s^{-2}
	(c)	41.8°
2	1.4 m s^{-1}	
3	**(a)**	10 J
	(b)	0.27 m
4	**(a)**	0.15 kg m^2
	(b)	11 rad s^{-1}
5	0.054 kg m^2	
6	4.63 rad s^{-2}	
7	2.51×10^{-4} kg m^2	
8	**(a)**	1.7 m s^{-2}
	(b)	6.9 m s^{-2}

Worked Solution

1 (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2)\cos\theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2}I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2)\cos\theta + \frac{1}{2}I\omega^2 \Rightarrow \omega^2 = (MgH/I)(1-\cos\theta).$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 10-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1-\cos\theta)} = \sqrt{\frac{3(9.80\ \text{m/s}^2)}{55.0\ \text{m}}(1-\cos 35.0°)} = 0.311\ \text{rad/s}.$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so

$$a_r = 3g\,(1-\cos\theta) = 3\,(9.80\ \text{m/s}^2)(1-\cos 35.0°) = 5.32\ \text{m/s}^2.$$

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 10-1 since the acceleration is not uniform. Hence, we differentiate

$$\omega^2 = (3g/H)(1-\cos\theta)$$

with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \Rightarrow \alpha = (3g/2H)\sin\theta.$$

Consequently,

$$a_t = H\alpha = \frac{3g}{2}\sin\theta = \frac{3(9.80\ \text{m/s}^2)}{2}\sin 35.0° = 8.43\ \text{m/s}^2.$$

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, $\sin\theta = 2/3$ and we obtain $\theta = 41.8°$.

2 The rotational inertia of the spherical shell is *2MR²/3,* so th kinetic energy (after the object descended distance *h* is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega^2_{\text{sphere}} + \frac{1}{2}I\omega^2_{\text{pulley}} + \frac{1}{2}mv^2.$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy *mgh* with which the system started. We substitute *v*/*r* for the pulley's angular speed and *v*/*R* for that of the sphere and solve for *v*.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$

$$= \sqrt{\frac{2(9.8)(0.82)}{1 + 3.0\times10^{-3}/((0.60)(0.050)^2) + 2(4.5)/3(0.60)}} = 1.4 \text{ m/s}.$$

3 **THINK** As the box falls, gravitational force gives rise to a torque that causes the wheel to rotate.

EXPRESS We employ energy methods to solve this problem; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, where $K_{\text{box}} = m_{\text{box}}v^2/2$. The rotational kinetic energy of the wheel is $K_{\text{rot}} = I\omega^2/2$.

COE

ANALYZE (a) With $K_{\text{box}} = 0.60$ J, we find the speed of the box to be

same V_{tangent}

$v = r\omega$

$$K_{\text{box}} = \frac{1}{2}m_{\text{box}}v^2 \Rightarrow v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = \sqrt{\frac{2(6.0 \text{ J})}{6.0 \text{ kg}}} = 1.41 \text{ m/s},$$

same string.

implying that the angular speed is ω = (1.41 m/s)/(0.20 m) = 7.07 rad/s. Thus, the kinetic energy of rotation is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.40 \text{ kg}\cdot\text{m}^2)(7.07 \text{ rad/s})^2 = 10.0 \text{ J}.$$

(b) Since it was released from rest, we will take the initial position to be our reference point for gravitational potential. Energy conservation requires

$$K_0 + U_0 = K + U \quad \Rightarrow \quad 0 + 0 = (6.0\text{ J} + 10.0\text{ J}) + m_{\text{box}} g(-h).$$

Therefore,

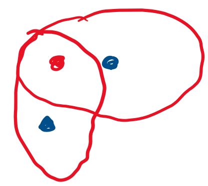
$$h = \frac{K}{m_{\text{box}} g} = \frac{6.0\text{ J} + 10.0\text{ J}}{(6.0\text{ kg})(9.8\text{ m/s}^2)} = 0.27\text{ m}.$$

LEARN As the box falls, its gravitational potential energy gets converted into kinetic energy of the box as well as rotational kinetic energy of the wheel; the total energy remains conserved.

4 (a) We use the parallel-axis theorem to find the rotational inertia:

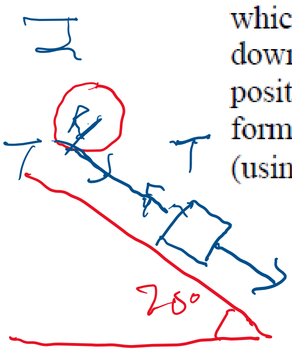
$$\boxed{I = I_{\text{com}} + Mh^2} = \frac{1}{2}MR^2 + Mh^2 = \frac{1}{2}(20\text{ kg})(0.10\text{ m})^2 + (20\text{ kg})(0.50\text{ m})^2 = 0.15\text{ kg}\cdot\text{m}^2.$$

(b) Conservation of energy requires that $\underline{Mgh} = \frac{1}{2}I\omega^2$, where ω is the angular speed of the cylinder as it passes through the lowest position. Therefore,



$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20\text{ kg})(9.8\text{ m/s}^2)(0.050\text{ m})}{0.15\text{ kg}\cdot\text{m}^2}} = 11\text{ rad/s}.$$

5 We choose positive coridnate direction so that each is accelerating positively, which will allow us to set $a_{\text{box}} = R\alpha$ (for simplicity, we denote this as a). Thus, we choose downhill positive for the m = 2.0 kg box and (as is conventional) counterclockwise for positive sense of wheel rotation. Applying Newton's second law to the box and (in the form of Eq. 10-45) to the wheel, respectively, we arrive at the following two equations (using θ as the incline angle 20°, not as the angular displacement of the wheel).



mg sin θ

N2L

$$mg\sin\theta - T = ma$$

T=Iα

$$TR = I\alpha$$

a = rα

same string: same a

Since the problem gives a = 2.0 m/s², the first equation gives the tension $T = m(g\sin\theta - a)$ = 2.7 N. Plugging this and R = 0.20 m into the second equation (along with the fact that $\alpha = a/R$) we find the rotational inertia

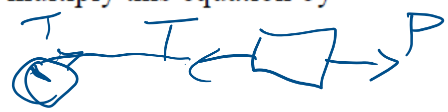
$$I = TR^2/a = 0.054\text{ kg}\cdot\text{m}^2.$$

(a) = rα
same

6 **THINK** The applied force P accelerates the block. In addition, it gives rise to a torque that causes the wheel to undergo angular acceleration.

EXPRESS We take rightward to be positive for the block and clockwise negative for the wheel (as is conventional). With this convention, we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P - T = ma$, where T is the tension in the cord. Similarly, applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$. Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \Rightarrow T = a\frac{I}{R^2}.$$



Adding this to the above equation (for the block) leads to $P = (m + I/R^2)a$. Thus, the angular acceleration is

$$\alpha = -\frac{a}{R} = -\frac{P}{(m + I/R^2)R}$$

$a = r\alpha$

same string.

ANALYZE With $m = 2.0$ kg, $I = 0.050\ \text{kg}\cdot\text{m}^2$, $P = 3.0$ N and $R = 0.20$ m, we find

$$\alpha = -\frac{P}{(m + I/R^2)R} = -\frac{3.0\ \text{N}}{[2.0\ \text{kg} + (0.050\ \text{kg}\cdot\text{m}^2)/(0.20\ \text{m})^2](0.20\ \text{m})} = -4.62\ \text{rad/s}^2,$$

or $|\alpha| = 4.62\ \text{rad/s}^2$.

LEARN The greater the applied force P, the greater the (magnitude of) angular acceleration. Note that the negative sign in α should not be mistaken for a deceleration; it simply indicates the clockwise sense to the motion.

7 Combining $\cancel{K = \frac{1}{2}I\omega^2}$ and $\tau_{net} = I\alpha$ we have $RF = I\alpha$, where α is given by $\frac{\omega}{t}$ (since $\omega_0 = 0$). We also use the fact that

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$$I = I_{\text{plate}} + I_{\text{disk}}$$

$\boxed{\tau = I\alpha}$

where $I_{\text{disk}} = \frac{1}{2}MR^2$ (item (c) in Table 10-2). Therefore,

FR

$$I_{\text{plate}} = \frac{RFt}{\omega} - \frac{1}{2}MR^2 = 2.51 \times 10^{-4}\ \text{kg}\cdot\text{m}^2.$$

don't care

$\alpha = \frac{d\omega}{dt}$

$= \frac{\omega - \omega_0}{t}$ $\omega_0 = 0$

$\alpha = \frac{\omega}{t}$

8 With counterclockwise positive, the angular acceleration α for both masses satisfies

$$\tau = mgL_1 - mgL_2 = I\alpha = \left(mL_1^2 + mL_2^2\right)\alpha,$$

$\tau = I\alpha$

by combining Eq. 10-45 with Eq. 10-39 and Eq. 10-33. Therefore, using SI units,

$$\alpha = \frac{g(L_1 - L_2)}{L_1^2 + L_2^2} = \frac{(9.8\text{ m/s}^2)(0.20\text{ m} - 0.80\text{ m})}{(0.20\text{ m})^2 + (0.80\text{ m})^2} = -\,8.65\text{ rad/s}^2$$

$I = \Sigma(mr^2)$

where the negative sign indicates the system starts turning in the clockwise sense. The magnitude of the acceleration vector involves no radial component (yet) since it is evaluated at $t = 0$ when the instantaneous velocity is zero. Thus, for the two masses, we apply Eq. 10-22:

$a_t = r\alpha$

(a) $|\vec{a}_1| = |\alpha|L_1 = \left(8.65\text{ rad/s}^2\right)\left(0.20\text{ m}\right) = 1.7\text{ m/s}.$

(b) $|\vec{a}_2| = |\alpha|L_2 = \left(8.65\text{ rad/s}^2\right)\left(0.80\text{ m}\right) = 6.9\text{ m/s}^2.$