Forms of Quadratic Functions

General Form

$y = ax^2 + bx + c$

Factorised Form

$y = a(x \pm p)(x \pm q)$

Completed square form

$y = a(x \pm h)^2 \pm k$

HOW?

e.g. express $2x^2 - 6x + 5$ in the form $a(x-h)^2 \pm k$.

1. ensure that coefficient of x^2 is 1.

$2(x^2 - 3x + \frac{5}{2})$ OR $2(x^2 - 3x) + 5$

2. To complete $x^2 \pm bx \pm c = (x \pm \frac{b}{2})^2 \pm c - (\frac{b}{2})^2$

$2\left[(x - \frac{3}{2})^2 + \frac{5}{2} - (-\frac{3}{2})^2\right]$

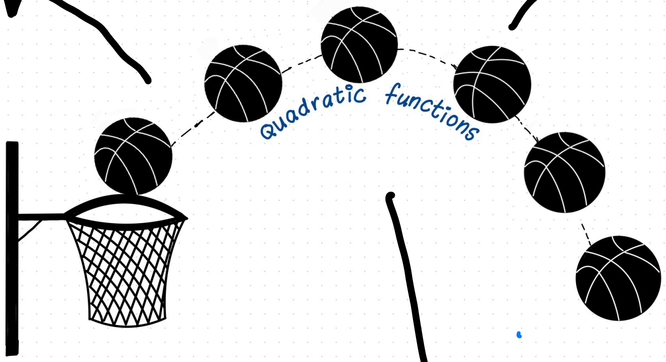
$= 2\left[(x - \frac{3}{2})^2 + \frac{1}{4}\right]$

$= 2(x - \frac{3}{2})^2 + \frac{1}{2}$ #

$= 2\left[(x - \frac{3}{2})^2 - (\frac{3}{2})^2\right] + 5$

$= 2(x - \frac{3}{2})^2 - \frac{9}{2} + 5$

$= 2(x - \frac{3}{2})^2 + \frac{1}{2}$



Turning Point

For graph of $y = a(x \pm h)^2 \pm k$,

- when $a > 0$,
  min turning point $= (\mp h, \pm k)$

- when $a < 0$,
  max turning point $= (\mp h, \pm k)$

e.g. $y = 2(x-3)^2 - 5$

Sol^n: min turning pt $=$ $(3, -5)$

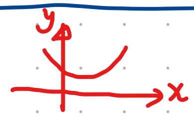
e.g. $y = -7(x+5)^2 + 8$

solution

max turning pt $=$ $(-5, 8)$

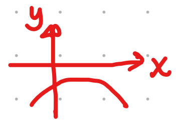
Conditions For Curve To Lie Completely Above OR Below x-axis

1. Above x-axis



- when $a > 0$
- minimum value of curve is positive

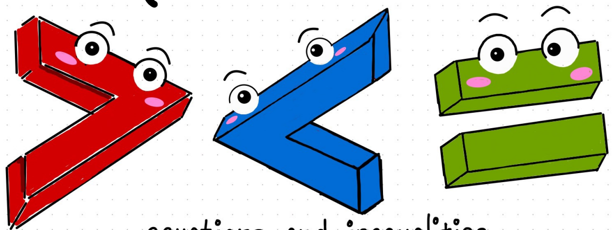
2. Below x-axis



- when $a < 0$
- maximum value of curve is negative

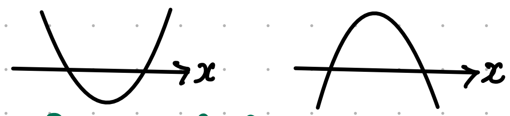
Solving Quadratics Equations

1. By Factorisation
2. By completing the square
3. By quadratic formula i.e. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$


equations and inequalities

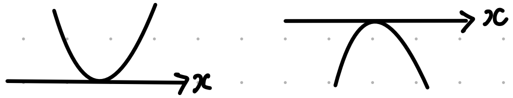
Nature of Roots

Case 1: 2 real and distinct roots



$b^2-4ac \; (D) > 0$

Case 2: 2 equal and real roots



$b^2-4ac \; (D) = 0$

Case 3: No real roots

$b^2-4ac \; (D) < 0$

$(y>0)$
- graph is always positive and lie completely above x-axis.

$(y<0)$
- graph is always negative and lie completely below x-axis

Case 4
- Real Roots
$b^2-4ac \geq 0$

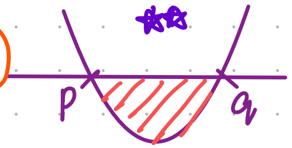
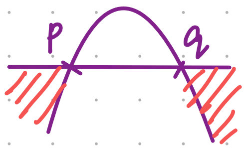
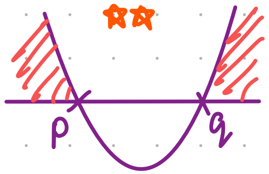
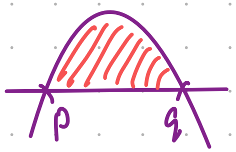
Case 5
- Graph is (a) non-positive $y=0$, $y<0$ (b) non-negative $y=0$, $y>0$

$b^2-4ac \leq 0$

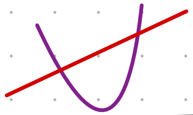
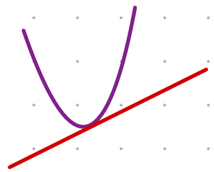
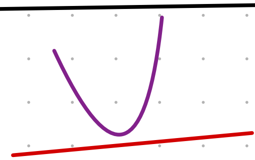
Solving Quadratics Inequalities

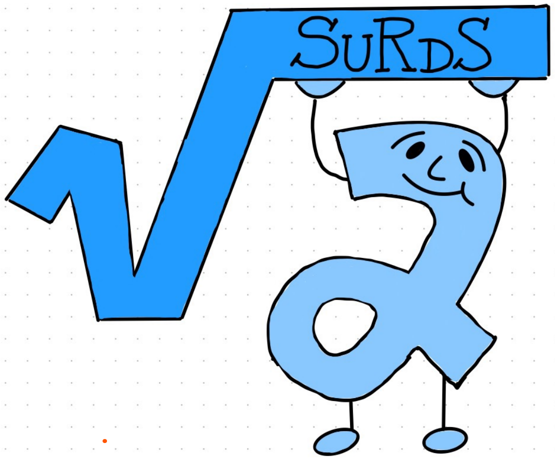
1. ensure that one side of the ~~equation~~ inequality is equal to ZERO (RHS)
2. Factorise quadratic expression.
3. Sketch a diagram & shade required region.

MdmSoh's class ☺

$ax^2+bx+c<0 \Rightarrow (\quad)(\quad)<0$	
 ★★ $\boxed{a>0}$ sol^n: $p<x<q$	 $a<0$ sol^n: $x<p$ OR $x>q$
$ax^2+bx+c>0 \Rightarrow (\quad)(\quad)>0$	
 ★★ $a>0$ sol^n: $x<p$ or $x>q$	 $a<0$ sol^n: $p<x<q$

Intersection btw line & curve

1. Line meets (cuts) curve at 2 points $\therefore b^2-4ac>0$	
2. Line meets (touches) curve at 1 point (line tangent to curve) $\therefore b^2-4ac=0$	
3. Line does not intersect curve. $\therefore b^2-4ac<0$	



LAWS OF SURDS

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
2. $\sqrt{a} \times \sqrt{a} = a$
3. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
4. $p\sqrt{a} \pm q\sqrt{a} = (p \pm q)\sqrt{a}$

SIMPLIFYING SURDS

e.g. $\sqrt{50} = \sqrt{25 \times 2}$

$= \sqrt{25} \times \sqrt{2}$

$= 5\sqrt{2}$ #

largest perfect SQ

eg. $2\sqrt{27} = 2(\sqrt{9 \times 3})$

$= 2(\sqrt{9} \times \sqrt{3}) = 2(3\sqrt{3})$

$= 6\sqrt{3}$ #

$= 2 \times 3\sqrt{3}$

$= 6\sqrt{3}$

RATIONALIZING SURDS

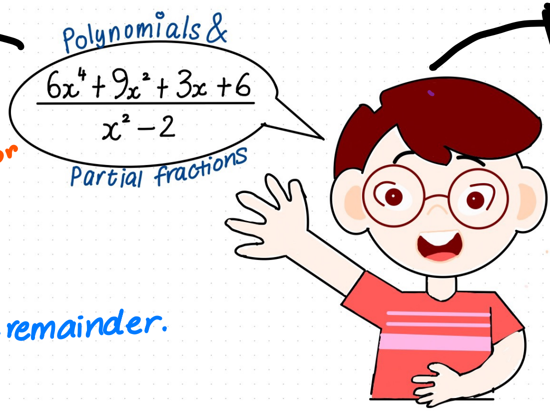
- you rationalize when the denominator is in surd form.

HOW? multiply with its conjugate surd.

e.g.

1. $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
2. $\frac{1}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{5(3)} = \frac{\sqrt{3}}{15}$
3. $\frac{2}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{2(1-\sqrt{2})}{(1)^2-(\sqrt{2})^2} = \frac{2-2\sqrt{2}}{1-2} = \frac{2-2\sqrt{2}}{-1}$

   $= -2+2\sqrt{2}$ #
4. $\frac{1}{5\sqrt{3}-2\sqrt{7}} \times \frac{5\sqrt{3}+2\sqrt{7}}{5\sqrt{3}+2\sqrt{7}} = \frac{5\sqrt{3}+2\sqrt{7}}{5(3)-2(7)} = 5\sqrt{3}+2\sqrt{7}$ #



Degree of a Polynomial

- the highest power in the polynomial

$\boxed{f(x) = \text{divisor} \times \text{quotient} + \text{remainder}}$ (degree < divisor)

$= D(x) \times Q(x) + R(x)$

Remainder Theorem

$f(x)$ divided by a linear divisor $(x-a)$, then $f(a)$ = remainder.

Factor Theorem

$f(x)$ divided by a linear factor $(x-a)$, then $f(a) = 0$.

e.g. A cubic equation $f(x)=0$ has roots $-3, -1$ & 4. Given that $f(x)$ leaves a remainder of 12 when it is divided by $(x+2)$, find $f(x)$ in descending powers of x & the remainder when $f(x)$ is divided by x^2-2x+1.

Solution

since $x=-3$, $x=-1$, $x=4$ are roots,

$\therefore f(x) = k(x+3)(x+1)(x-4)$ → By Factor Theorem,

By Remainder Theorem,

since $f(-2) = 12$

$k(1)(-1)(-6) = 12$

$6k = 12$

$k = 2$

$\therefore f(x) = 2(x+3)(x+1)(x-4)$

$= 2(x+3)(x^2-3x-4)$

$= 2(x^3-3x^2-4x+3x^2-9x-12)$

$= 2(x^3-13x-12)$

$= 2x^3-26x-24$

By long division:

$$
\begin{array}{r|l}
 & \quad 2x+4 \\
\hline
x^2-2x+1 & 2x^3+0x^2-26x-24 \\
 & -(2x^3-4x^2+2x) \\
\hline
 & \quad 4x^2-28x-24 \\
 & \quad -(4x^2-8x+4) \\
\hline
 & \quad\quad -20x-28
\end{array}
$$

Thus, remainder $= -20x-28$ #

Partial Fractions

① check whether $\frac{f(x)}{g(x)}$ is a * proper fraction.

* degree $f(x)$ < degree $g(x)$.

If degree $f(x) \geq$ degree $g(x)$, use long division to make it proper first.

② check whether you can completely factorise $g(x)$.

③ Express proper algebraic fractions according to the cases below.

Case	Denominator $g(x)$	Fractions	Partial Fractions
1	2 distinct & linear factors	$\frac{px+q}{(ax+b)(cx+d)}$	$\frac{A}{ax+b} + \frac{B}{cx+d}$
2	Repeated linear factors	$\frac{px+q}{(ax+b)^2}$	$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$
3	Linear & quadratic factor (can't be factorised)	$\frac{px+q}{(ax+b)(cx^2+d)}$	$\frac{A}{ax+b} + \frac{Bx+C}{(cx^2+d)}$

④ solve for unknown constants by substituting suitable values of x or comparing coefficients.

Solving Cubic Equations

① Find the first linear factor by trial and error.

② Find the remaining quadratic factor using LONG DIVISION

③ Cross-factorise the quadratic formula & solve.

* if quadratic factor cannot be factorised, use QUADRATIC FORMULA to solve it.

Binomial Theorem

$$(a+b)^n = \underbrace{{}^nC_0 (a)^{n-0} (b^0)}_{\text{1st term}} + {}^nC_1 (a)^{n-1} (b) + {}^nC_2 (a)^{n-2} (b)^2 + \dots + \underbrace{{}^nC_n (a)^{n-n} (b)^n}_{\text{last term}}$$

General Term Formula

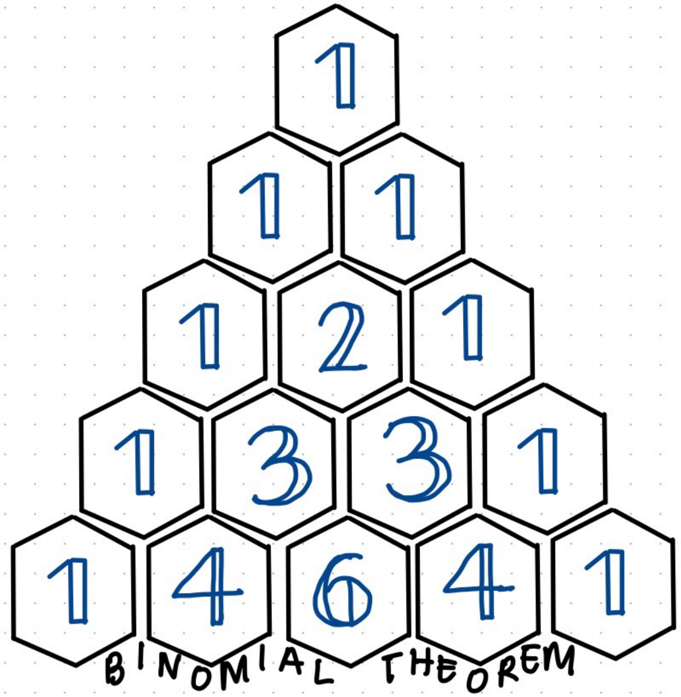
$$T_{r+1} = {}^nC_r (a)^{n-r} (b)^r$$

Factorial

$0! = 1$ and $n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$

$$\therefore {}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} =$$

- the term independent of x $\Rightarrow$ constant term
  $\Rightarrow$ power of $x = 0$.



eg.

Find the term independent of x in $\left(x + \frac{1}{2x}\right)^8$.

solution

General Formula : $\binom{8}{r}(x)^{8-r}\left(\frac{1}{2x}\right)^r = \binom{8}{r}(x)^{8-r}\left(\frac{1}{2}\right)^r\left(\frac{1}{x}\right)^r$

$= \binom{8}{r}(x)^{8-r}\left(\frac{1}{2}\right)^r (x^{-1})^r$

$= \binom{8}{r}\left(\frac{1}{2}\right)^r (x)^{8-r}(x^{-r})$

$= \underbrace{\binom{8}{r}\left(\frac{1}{2}\right)^r}_{\text{coefficient}} (x^{8-2r})$

since term independent of $x \Rightarrow x^0$

$\therefore x^{8-2r} = x^0$

$8 - 2r = 0$

$r = 4$

sub $r = 4$ into coefficient.

$\binom{8}{4}\left(\frac{1}{2}\right)^4 = \frac{35}{8}$ #.

LAWS OF INDICES

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$
- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

examples

e.g. Solve $9^x + 2(3^x) = 3^{x+2} - 12$

solution

$(3^2)^x + 2(3^x) = (3^x)(3^2) - 12$

$(3^x)^2 + 2(3^x) = (3^x)(3^2) - 12$

Let y be 3^x.

$y^2 + 2y = 9y - 12$

$y^2 - 7y + 12 = 0$

$(y-3)(y-4) = 0$

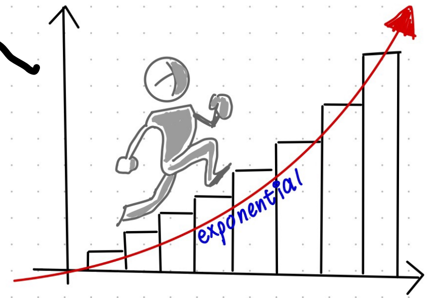
$y = 3$ or $y = 4$

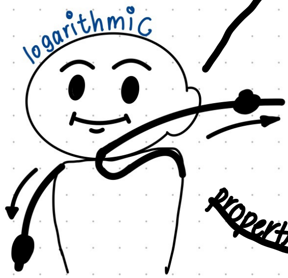
$3^x = 3$; $3^x = 4$

$x = 1$ #

$x \ln 3 = \ln 4$

$x = \frac{\ln 4}{\ln 3} = 1.26185\ldots = 1.26$ (3sf) #





$y = \ln(x)$

$$\boxed{y = a^x \Longleftrightarrow x = \log_a y}$$

LAWS OF LOGARITHMS

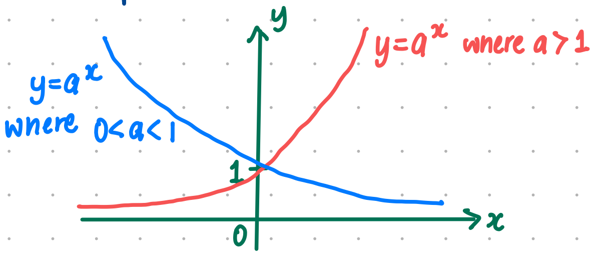
- $\log_a x + \log_a y = \log_a(xy)$ [PRODUCT LAW]
- $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$ [QUOTIENT LAW]
- $\log_a x^r = r\log_a x$ [POWER LAW]

properties

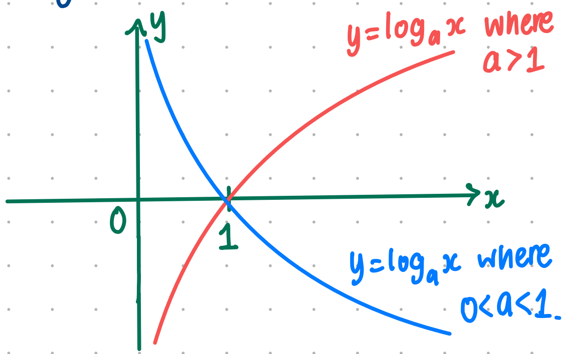
- $\log_a a = 1$
- $\log_a 1 = 0$
- $\lg y = \log_{10} y$
- $\ln y = \log_e y$
- $\log_a b = \frac{\log_c b}{\log_c a}$ [change of BASE]

GRAPHS

- exponential



- logarithmic



examples

e.g. Solve

(a) $\log_5 x = 4\log_x 5 + 3$

solution

$\log_5 x = 4\left[\frac{\log_5 5}{\log_5 x}\right] + 3$

$\log_5 x = 4\left[\frac{1}{\log_5 x}\right] + 3$

Let y be $\log_5 x$.

$\therefore y = 4\left(\frac{1}{y}\right) + 3$

$y^2 = 4 + 3y$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$\therefore y = 4$ or $y = -1$

$\log_5 x = 4$; $\log_5 x = 1$

$x = 5^4$; $x = 5^1$

$x = 625$ # ; $x = 5$ #

(b) $\log_5(5-4x) = \log_{\sqrt{5}}(2-x)$

solution

$\log_5(5-4x) = \left[\frac{\log_5(2-x)}{\log_5\sqrt{5}}\right]$

$\log_5(5-4x) = \frac{\log_5(2-x)}{\log_5 5^{\frac{1}{2}}}$

$\log_5(5-4x) = \frac{\log_5(2-x)}{\frac{1}{2}}$

$\log_5(5-4x) = 2\log_5(2-x)$

$\log_5(5-4x) = \log_5(2-x)^2$

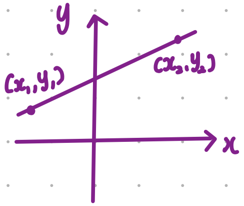
$\therefore (5-4x) = (2-x)^2$

$5 - 4x = 4 - 4x + x^2$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$\therefore x = 1$ # or $x = -1$ #



Straight line graphs

- **GRADIENT**

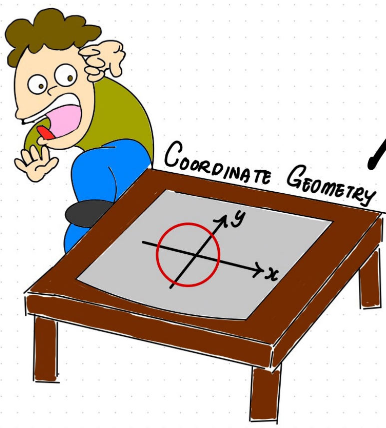
$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

- **LENGTH**

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- **MIDPOINT**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Circles

STANDARD FORM

$(x-a)^2 + (y-b)^2 = r^2$

where r = radius; centre = (a, b)

GENERAL FORM

$x^2 + y^2 + 2gx + 2fy + c = 0$

where centre = $(-g, -f)$

radius = $\sqrt{g^2 + f^2 - c}$

- **PARALLEL LINES**

If l_1 is parallel to l_2, then their gradients are EQUAL.

$$\boxed{m_1 = m_2}$$

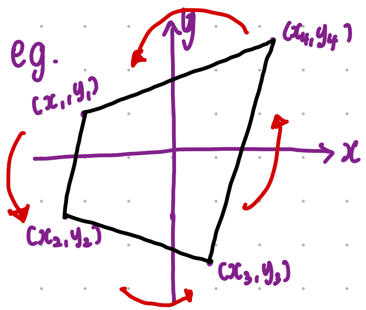
- **PERPENDICULAR LINES**

If l_1 is perpendicular to l_2, then the product of gradients $= -1$.

$$\boxed{m_1 \times m_2 = -1}$$

- **AREA OF RECTILINEAR FIGURE**

eg.



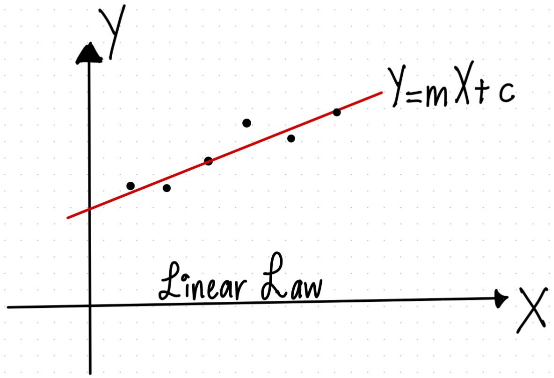
- must go in anticlockwise direction
- must end with the starting coordinates.

$\boxed{\text{solution}}$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\left[(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_1y_4 + x_4y_3 + x_3y_2 + x_2y_1)\right]$$

change to SQUARE bracket once you perform ↘ sum & ↙ sum



THINGS TO TAKE NOTE

- variables X and Y cannot contain unknown constants such as a and b.
- constants m and c cannot contain variables x and y.

LINEARISING NON-LINEAR FUNCTION

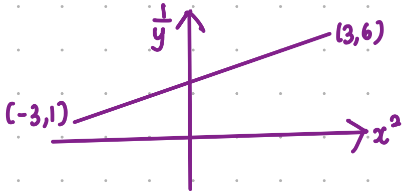
e.g. Express $y = ab^x$ in a linear form $Y = mX + c$.

$\boxed{\text{Solutions}}$

Take lg on both sides.

$$\lg y = \lg (ab^x)$$
$$\lg y = \lg a + \lg b^x$$
$$\lg y = \lg a + x \lg b$$
$$\underbrace{\lg y}_{Y} = \underbrace{\lg b}_{m}\,\underbrace{(x)}_{X} + \underbrace{\lg a}_{c}$$

e.g. The diagram shows a st. line obtained by plotting $\frac{1}{y}$ against x^2. Express y in terms of x.



$\boxed{\text{Solution}}$

① Find gradient. $m = \frac{6-1}{3-(-3)} = \frac{5}{6}$

② Find y-intercept. $Y = mX + c$

$$6 = \frac{5}{6}(3) + c$$
$$c = \frac{7}{2}$$
$$\therefore Y = \frac{5}{6}X + \frac{7}{2}$$

③ sub $Y = \frac{1}{y}$ and $X = x^2$,

$$\therefore \frac{1}{y} = \frac{5}{6}x^2 + \frac{7}{2}$$
$$\frac{1}{y} = \frac{5x^2 + 21}{6}$$
$$\therefore y = \frac{6}{5x^2 + 21}$$

Trigonometric Functions and Graphs

Special angles

Trigo	$30°(\frac{\pi}{6})$	$45°(\frac{\pi}{4})$	$60°(\frac{\pi}{3})$
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Definitions

Amplitude (a)

half the difference of the maximum and minimum value.

$c = \frac{1}{2}(\max + \min)$

$a = \frac{1}{2}(\max - \min)$

Period (b)

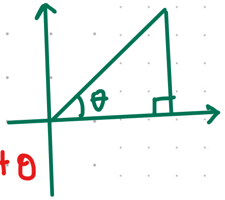
angle measured for one complete cycle

Complementary angles

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \frac{1}{\tan\theta} = \cot\theta$

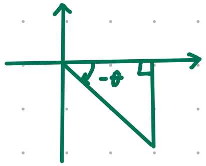


Negative angles

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$



Sin θ = +ve (2nd)	All positive (1st)
Tan θ = +ve (3rd)	Cos θ = +ve (4th)

(axes: y, x)

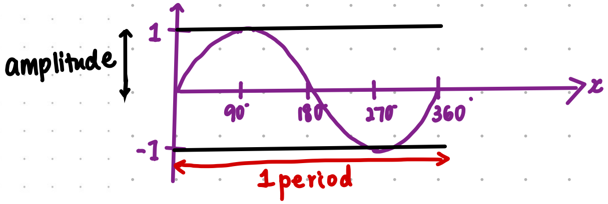
1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$
4. $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

- For $y = a\sin(bx) + c$
  $y = a\cos(bx) + c$
  - amplitude $= a$
    period $= \frac{360°}{b}$
  - max $= a + c$; min $= -a + c$
- For $y = a\tan(bx) + c$
  - amplitude = NIL
    period $= \frac{180°}{b}$
- c = positive; graph shifts upwards
- c = negative; graph shifts downwards

Basic Trigo Graphs

Sine Graph

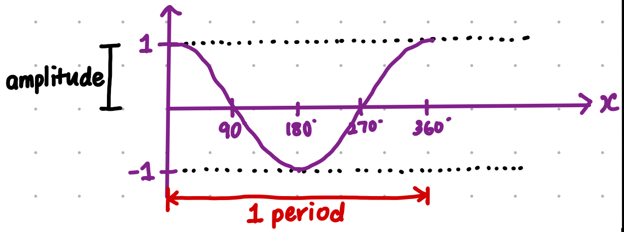
$y = \sin x$ for $0° \le x \le 360°$



Amplitude = 1
Period = 360°
Max = 1 ; Min = -1
Range: $-1 \le \sin x \le 1$

Cosine Graph

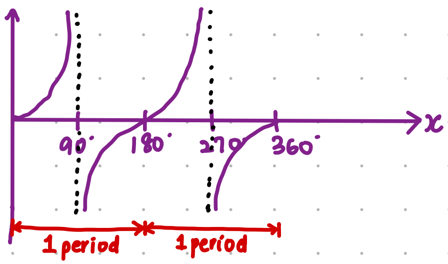
$y = \cos x$ for $0° \le x \le 360°$



Amplitude = 1
Period = 360°
Max = 1 ; Min = 1
Range : $-1 \le x \le 1$

Tangent Graph

$y = \tan x$ for $0° \le x \le 360°$



No amplitude
Period = 180°
No max ; No min

TRIGO EQUATIONS

AND IDENTITIES

SOLVING TRIGO EQUATIONS

SOLVING TRIGO EQUATIONS

① Express equation in the form of sin, cos or tan.
(i.e. $\sin(n\theta) = a$; $\cos(n\theta) = a$, $\tan(n\theta) = a$)

② Calculate the basic angle, α. (acute ∡) * using positive value of a.
(i.e. $\alpha = \sin^{-1}(a)$; $\alpha = \cos^{-1}(a)$; $\alpha = \tan^{-1}(a)$)

③ Identify which quadrants it lies.

Sin	All
Tan	Cos

④ check the range of new angles $(n\theta)$.

⑤ Find angle $(n\theta)$ and solve for θ.

E.g. Solve $\operatorname{cosec}(x-10°) = 2.5$ for $0° < x < 270°$

STEP 1 $\dfrac{1}{\sin(x-10°)} = \dfrac{5}{2}$

$\sin(x-10°) = 0.4$

STEP 2 ref ∡ $\alpha = \sin^{-1}(0.4)$
$= 23.578°$

STEP 3 lies in 1st & 2nd quad.

STEP 4 $-10° < x-10° < 260°$ (new range)

STEP 5 $x-10° = 23.578°,\ 180°-23.578°,$

$x = 33.578°,\ 166.422°$

$= 33.6°,\ 166.4°$ (1dp)

PRINCIPAL VALUES

$\left(-\frac{\pi}{2}\right)$ $-90° \le \sin^{-1}x \le 90°$ $\left(\frac{\pi}{2}\right)$, where $-1 \le x \le 1$

(0) $0° \le \cos^{-1}x \le 180°$ (π), where $-1 \le x \le 1$

$\left(-\frac{\pi}{2}\right)$ $-90° < \tan^{-1}x < 90°$ $\left(\frac{\pi}{2}\right)$, where x is any real number

e.g. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

$\sin 30° = \frac{1}{2}$

$\sin(-30°) = -\frac{1}{2}$

$\therefore$ principal value $= -30°$

ADDITION FORMULAE

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

DOUBLE ANGLE FORMULA

- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
  $= 1 - 2\sin^2 A$
  $= 2\cos^2 A - 1$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

e.g. Given that $\sin A = \frac{3}{5}$ (1st, 2nd) and $\tan A > 0$, (1st, 3rd) find the value of

(a) $\cos 2A$ (b) $\cos \frac{A}{2}$

(a) $\cos 2A = 1 - 2\sin^2 A$
$= 1 - 2\left[\frac{3}{5}\right]^2$
$= -\frac{1}{5}$

(b) $\cos 2\left(\frac{A}{2}\right) = 2\cos^2\left(\frac{A}{2}\right) - 1$

$\cos A = 2\cos^2\left(\frac{A}{2}\right) - 1$

$\frac{4}{5} = 2\cos^2\left(\frac{A}{2}\right) - 1$

$\frac{9}{5} = 2\cos^2\left(\frac{A}{2}\right)$

$\frac{9}{10} = \cos^2\left(\frac{A}{2}\right)$

$\cos\left(\frac{A}{2}\right) = \frac{3}{\sqrt{10}}$ (rej $-\frac{3}{\sqrt{10}}$)

$= \frac{3\sqrt{10}}{10}$

R - FORMULAE

- $a \sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$
- $a \cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$

where

$R = \sqrt{a^2+b^2}$ and $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

How to determine maximum and minimum volume?

- For $a\sin\theta \pm b\cos\theta$,
  Max value $= \sqrt{a^2+b^2}$, when $\sin(\theta \pm \alpha) = 1$.
  Min value $= -\sqrt{a^2+b^2}$, when $\sin(\theta \pm \alpha) = -1$.

- For $a\cos\theta \pm b\sin\theta$,
  Max value $= \sqrt{a^2+b^2}$, when $\cos(\theta \mp \alpha) = 1$
  Min value $= -\sqrt{a^2+b^2}$, when $\cos(\theta \mp \alpha) = -1$.

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

ALGEBRA

- $\frac{d}{dx}(a) = 0$, where a is a constant
- $\frac{d}{dx}(ax^n) = nax^{n-1}$
- $\frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1}(a)$
- $\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$
- $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

differentiation rules

$\frac{d}{dx}(ax+b)$

TRIGONOMETRIC

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}[\sin(ax+b)] = \cos(ax+b)(a)$	$\frac{d}{dx}[\sin^n x] = n[\sin x]^{n-1}[\cos x]$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}[\cos(ax+b)] = -\sin(ax+b)(a)$	$\frac{d}{dx}[\cos^n x] = n[\cos x]^{n-1}[-\sin x]$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}[\tan(ax+b)] = \sec^2(ax+b)(a)$	$\frac{d}{dx}[\tan^n x] = n[\tan x]^{n-1}[\sec^2 x]$

EXPONENTIAL

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$

LOGARITHMS

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

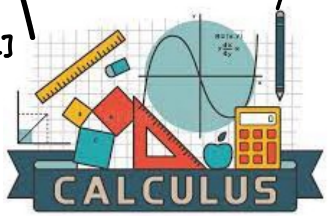
$\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$

INCREASING FUNCTION

$\frac{dy}{dx} > 0$

DECREASING FUNCTION

$\frac{dy}{dx} < 0$



rates of change

$$\boxed{\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}}$$

e.g. Eqn of curve is $y = \sqrt{2x^2-7}$. Given that x is changing at a constant rate of 0.25 units per second, find the rate of change of y when $x = 4$.

What's given? $\frac{dx}{dt} = 0.25$

Find? $\frac{dy}{dt}$?

$y = (2x^2-7)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(2x^2-7)^{-\frac{1}{2}}(4x)$

$= \frac{2x}{\sqrt{2x^2-7}}$

$\boxed{\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}}$

$= \frac{2x}{\sqrt{2x^2-7}} \times 0.25$

$= \frac{0.5x}{\sqrt{2x^2-7}}$

When $x = 4$,

$\frac{dy}{dt} = \frac{0.5(4)}{\sqrt{2(4^2)-7}}$

$= 0.4$ #.

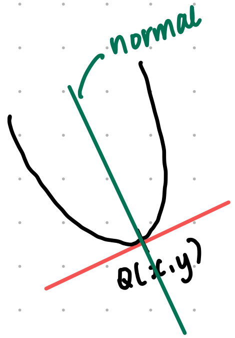
EQNS OF TANGENTS & NORMAL

definition of $\frac{dy}{dx}$

If $y = f(x)$ is the equation of curve, then $\frac{dy}{dx}$ = gradient function / tangent to the curve at any pt (x, y)

Finding gradient & eqn of tangent at pt Q

1. Diff. $y = f(x)$ to find $\frac{dy}{dx}$
2. sub x-coord of Q into $\frac{dy}{dx}$, to find the value of grad of tangent at Q. ($\square$)
3. sub value of gradient and coordinates of Q into eqn $y = mx + c$ to find c. ($\triangle$) $\boxed{y - y_1 = m(x - x_1)}$
4. Eqn of tangent is $y = \square x + \triangle$



Finding gradient & eqn of normal at pt Q

1. Diff. $y = f(x)$ to find $\frac{dy}{dx}$
2. sub x-coord of Q into $\frac{dy}{dx}$, to find the value of grad of tangent, b, at point Q.
   ↳ grad of normal $= -\frac{1}{b}$ ($\bullet$)
3. sub value of gradient of normal & coord. Q into eqn $y = mx + c$ to find c. ($\triangle$)
4. Eqn of tangent is $y = \bullet x + \triangle$

e.g. Find eqn of tangent and normal to the curve $y = 2x^2 + x$ at point R(−1, 1).

$\frac{dy}{dx} = 4x + 1$

when $x = -1$, $\frac{dy}{dx} = -3$ → grad tangent

$1 = -3(-1) + c$

$1 = 3 + c$

$c = -2$

∴ eqn of tangent ⇒ $y = -3x - 2$

grad normal $= -\frac{1}{3}$

$1 = -\frac{1}{3}(-1) + c$

$1 = \frac{1}{3} + c$

$c = \frac{2}{3}$

∴ eqn of normal ⇒ $y = -\frac{1}{3}x + \frac{2}{3}$

CALCULUS

STATIONARY POINTS

- occurs when $\frac{dy}{dx} = 0$.
- can be maximum, minimum or point of inflexion.

FIRST DERIVATIVE TEST

- to determine the nature of stat points

Let's say after solving for $\frac{dy}{dx}=0$; we get $x=x_0$.

Maximum Point

x	x_0^-	x_0	x_0^+
$\frac{dy}{dx}$	+ve	0	-ve
sketch	/	—	\

Minimum Point

x	x_0^-	x_0	x_0^+
$\frac{dy}{dx}$	-ve	0	+ve
sketch	\	—	/

Point of Inflexion

x	x_0^-	x_0	x_0^+
$\frac{dy}{dx}$	+ve	0	+ve
sketch	/	—	/

Point of Inflexion

x	x_0^-	x_0	x_0^+
$\frac{dy}{dx}$	-ve	0	-ve
sketch	\	—	\

SECOND DERIVATIVE TEST

- If $\frac{d^2y}{dx^2} < 0$ at $x=x_0$, then it is a max pt.
- If $\frac{d^2y}{dx^2} > 0$ at $x=x_0$, then it is a min pt.
- If $\frac{d^2y}{dx^2} = 0$ at $x=x_0$, then go back to 1st derivative test.

STATIONARY VALUE

- occurs when $\frac{dy}{dx}=0$.
- If $\frac{d^2y}{dx^2} < 0$ at $x=x_0$, then it is a max value.
- If $\frac{d^2y}{dx^2} > 0$ at $x=x_0$, then it is a min value.

e.g. A dot on a tablet screen moves along the curve $y = 2x^3 - 3x^2 - 36x + 5$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(ii) Find the minimum gradient of the tangent to the curve.

(i) $y = 2x^3 - 3x^2 - 36x + 5$

$$\frac{dy}{dx} = 8x^2 - 6x - 36$$

$$\frac{d^2y}{dx^2} = 16x - 6$$

(ii) Let z be the gradient of tangent to the curve.

$$z = \frac{dy}{dx} = 8x^2 - 6x - 36$$

$$\frac{dz}{dx} = 16x - 6$$

For stationary value, $\frac{dz}{dx} = 0$.

$$16x - 6 = 0$$

$$x = \frac{3}{8}$$

sub $x = \frac{3}{8}$ into z.

$$z = 8\left(\frac{3}{8}\right)^2 - 6\left(\frac{3}{8}\right) - 36$$

$$= -\frac{297}{8}$$

$$\frac{d^2z}{dx^2} = 16 > 0$$

The minimum gradient is $-\frac{297}{8}$.

Algebraic

$$\int a\,dx = ax + c$$

$$\int ax^n\,dx = \frac{ax^{n+1}}{x+1} + c$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)(a)} + c$$

where c is a constant

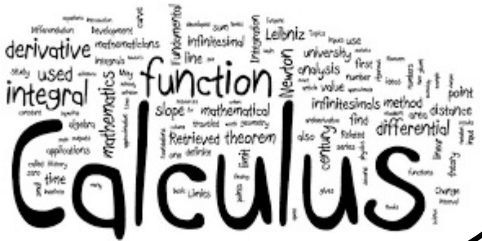
Exponential

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c$$

$$\int \frac{1}{x}\,dx = \ln x + c$$

$$\int \frac{1}{ax+b}\,dx = \frac{\ln(ax+b)}{a} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln[f(x)] + c$$



Trigonometry

$$\int \cos x\,dx = \sin x + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + c$$

$a \rightarrow \frac{d}{dx}(ax+b)$

$$\int \sin(ax+b)\,dx = \frac{-\cos(ax+b)}{a} + c$$

$$\int \sec^2(ax+b)\,dx = \frac{\tan(ax+b)}{a} + c$$

RULES OF DEFINITE INTEGRALS

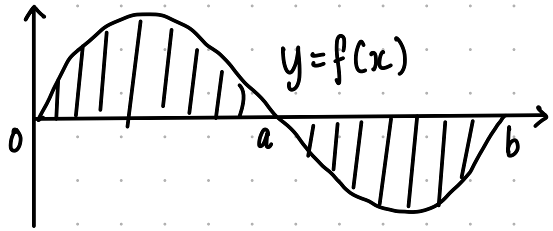
$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_b^c f(x)\,dx + \int_a^b f(x)\,dx.$$

AREA BOUNDED BY CURVE & X-AXIS



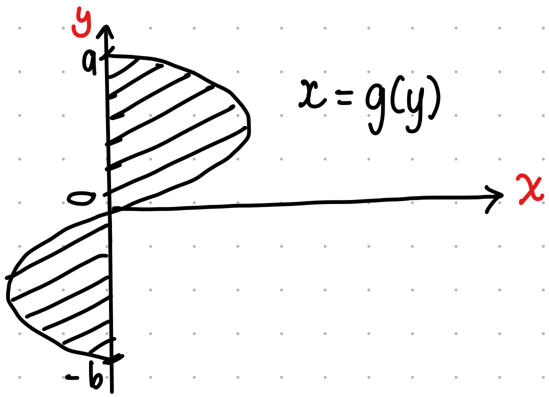
Total area of shaded region

$$= \int_0^a f(x)\,dx + \left|\int_a^b f(x)\,dx\right|$$

→ modulus

eg. $|-2| = 2$

AREA BOUNDED BY CURVE & Y-AXIS

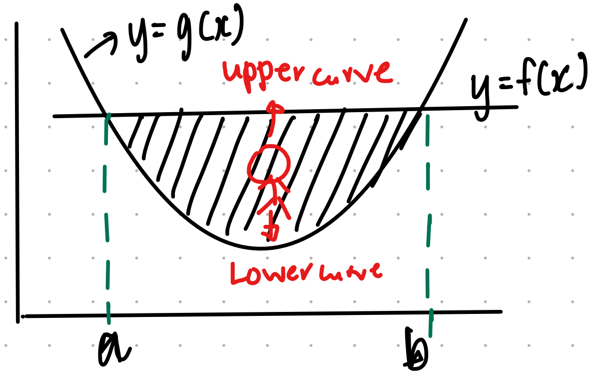


Total area of shaded region

$$= \int_0^a g(y)\,dy + \left|\int_{-b}^{0} g(y)\,dy\right|$$

→ modulus

AREA BOUNDED BY LINE & CURVE



Area of shaded region

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

OR $\displaystyle\int_a^b \underbrace{f(x)}_{\text{upper curve}} - \underbrace{g(x)}_{\text{Lower curve}}\,dx$

$$\text{Displacement}(s) \xrightarrow{v=\frac{ds}{dt}} \text{Velocity}(v) \xrightarrow{a=\frac{dv}{dt}} \text{Acceleration}(a)$$

$$\text{Acceleration}(a) \xrightarrow{v=\int a\,dt} \text{Velocity}(v) \xrightarrow{s=\int v\,dt} \text{Displacement}(s)$$

NOTATIONS

- displacement / distance (s)
- velocity / speed (v) ie. $\frac{ds}{dt}$
- acceleration (a) ie. $\frac{dv}{dt}$

KEY TERMS

① Initial ⇒ sub $t=0$

② Instantaneous rest ⇒ when $v=0$

③ Max / Min displacement ⇒ $\frac{ds}{dt}=0$

④ Max / Min velocity ⇒ $\frac{dv}{dt}=0$

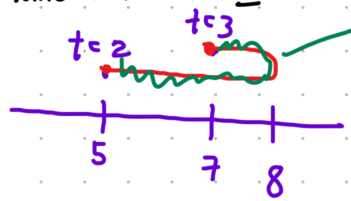
⑤ Distance travelled in the 3rd second (ie.)

↳ sub $t=3$ to find ~~distance~~ displacement.

↳ sub $t=2$ to find ~~distance~~ displacement.

∴ distance travelled 3rd second is

⑤



distance between the 2 pts

e.g. A particle moves in a straight line so that, t seconds after leaving a fixed point, O, its displacement, s cm, from O can be modelled by $s = t^3 - 9t^2 + 15t$.

(i) Find the values of t in which it is at instantaneous rest.

$$v = \frac{ds}{dt} = 3t^2 - 18t + 15$$

Instantaneous rest ⇒ $v = 0$. ∴ $3t^2 - 18t + 15 = 0$

$$3(t-1)(t-5) = 0$$

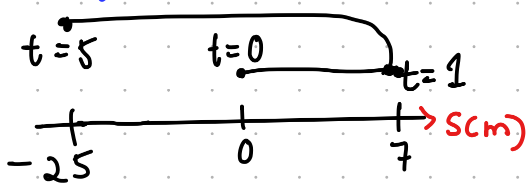
$$\therefore t = 1 \text{ or } 5 \text{ sec.}$$

(ii) Find acceleration of particle when it first comes to instantaneous rest.

$$a = \frac{dv}{dt} = 6t - 18$$

when $t = 1$, $a = -12 \text{ m/s}^2$

(iii) Find the total distance travelled from $t=0$ to $t=5$?



when $t=0$; $s = 0$

when $t=1$, $s = 7$

when $t = 5$, $s = -25$

total dist $= 7 + 25 + 7$

$= 39\text{m}.$

USING CONGRUENT & SIMILAR △S

congruency test

① side-side-side (SSS)
* 3 equal corresponding sides.

② side-angle-side (SAS)

③ angle-side-angle (ASA)

④ right angle-hypotenuse-side (RHS)

similarity test.

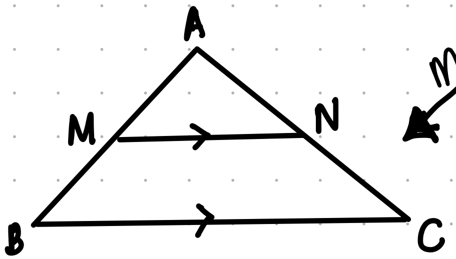
① Angle-Angle (AA)
- 2 corresponding angles equal

② side-side-side (SSS)
- 3 corresponding sides in the same ratio

③ side-side-angle (SSA)
- 2 corresponding sides in same ratio and 1 included angle equal.

PROOF
of Plane geometry

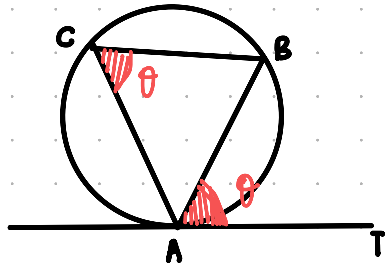
midpoint theorem



If M and N are midpoints of AB and AC respectively,
then,
① MN // BC
② BC = 2MN

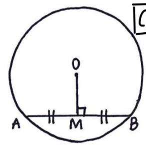
reason: midpt theorem

tangent chord theorem
OR Alt. seg Theorem



(CIRCLE PROPERTIES)

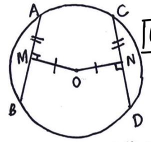
①



(⊥ bisector of chord)

* perpendicular bisector of a chord passes through the centre.
* line joining midpoint AB to centre of circle is perpendicular to chord

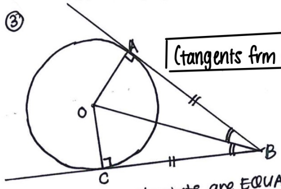
②



(equal chords)

* equal chords equidistant from centre. (If AB = CD, then OM = ON)
* chords are equidistant from centre are equal in length. (If OM = ON, then AB = CD)

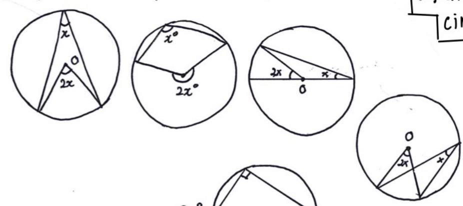
③



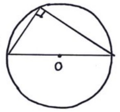
(tangents frm ext. pt)

* tangents from external points are EQUAL (AB = CB).
* lines from external point to centre of circle bisects angle.

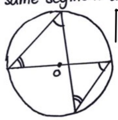
① Angle at centre is equal to twice angle at circumference (∡ at ⊙ = 2∡ at circumference)



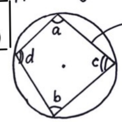
② Angle in a semicircle = 90°
(∡ in semicircle)



③ Angles in the same segment are EQUAL.
(∠ in same segment)



④ Angles in opposite segments add up to 180°.
(∡ in opp segment)



cyclic quadrilateral (all points on circumference of circle)

(∠a + ∠b = 180°) OR (∠c + ∠d = 180°)

⑤ Tangent to circle is perpendicular to radius. (tan ⊥ rad)