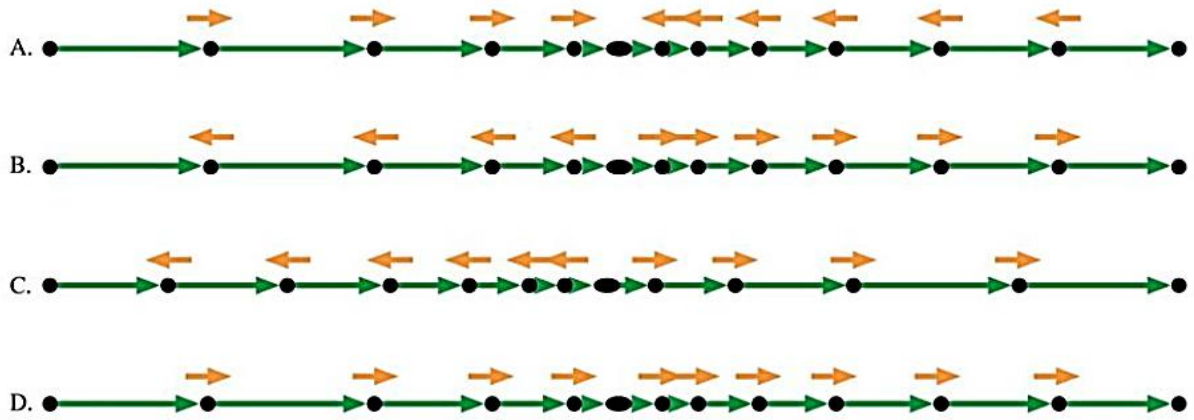
Answers to Examples, Quizzes & Mini-test [Kinematics]

Quiz

A cyclist riding at 7 m s^{-1} sees a stop sign and comes to a complete stop in 4 s. He then, in 6 s, returns to a speed of 5 m s^{-1}. Which of the following shows his motion diagram?



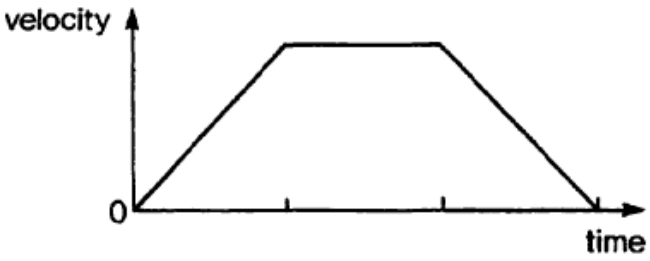
Solution:

Answer is either B or C based on the relative directions of the acceleration and velocity vectors. Since the cyclist is slowing down in the first part of his motion, the acceleration and velocity vectors should be in opposite directions. Since he speeds up after the stop, the acceleration and velocity vectors should be in same direction.

Answer is B not C because the final velocity is smaller than the initial velocity, so the last arrow should be shorter than the first arrow.

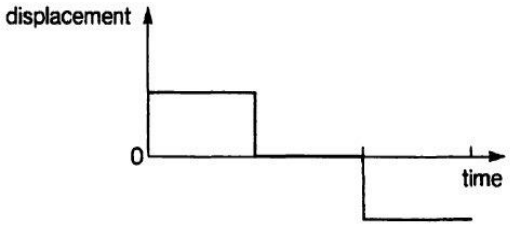
Quiz

The graph of velocity against time for a moving object is shown.

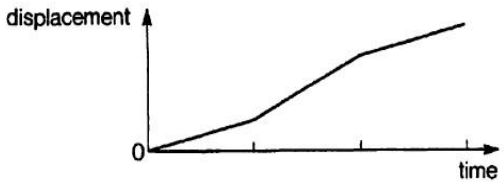


Which of the following is the corresponding graph of displacement against time? (N95/I/5)

A 

B 

C 

D 

Solution:

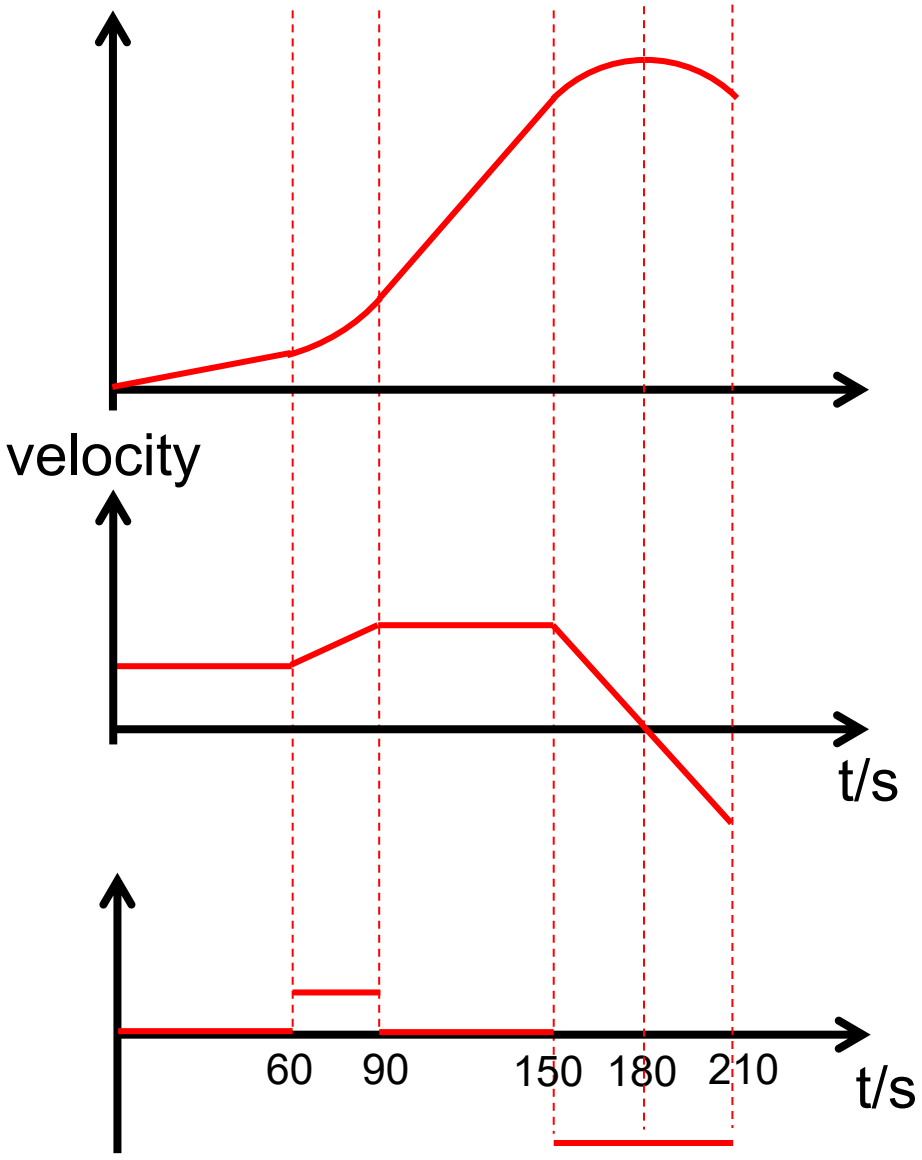
Answer: C

- Velocity is always +ve, hence gradient of s-t graph should be always +ve (i.e. should neither be -ve nor zero). Eliminate A & B.
- Velocity doesn't change abruptly, so eliminate D where 'sharp' points indicate abrupt change in gradient and thus abrupt changes in velocity.

MINI-TEST 1

A car is travelling along a straight road at a constant velocity initially for 1 minute. After that, the driver steps on the accelerator, causing the car's speed to increase uniformly for 30 s. It then travels at constant velocity again for another minute before the driver slows down uniformly for 30 s to a momentary stop and then makes a U-turn back with the same acceleration for 30 s. Sketch the displacement-time, velocity-time and acceleration-time graphs to illustrate the motion of the car.

My solution:



Quiz

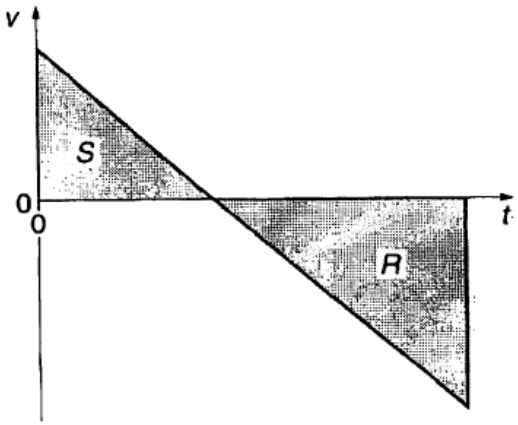
1) A ball is thrown vertically upwards. While the ball is in free fall, its acceleration
   A) increase,
   B) decrease
   C) remain constant
2) After a ball is thrown vertically upward and is in air, its speed
   A) increases,
   B) decreases,
   C) decreases and then increases,
   D) remains the same.

Solution:

Answer: Both (C)

Quiz

A stone is thrown upwards from the top of a cliff. After reaching its maximum height, it falls past the cliff-top and into the sea. The graph below shows how the vertical velocity v of the stone varies with time t after being thrown upwards. R and S are the magnitudes of the areas of the two triangles.

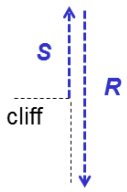


What is the height of the cliff-top above the sea? (N98/I/3)

A) R
B) S
C) R+ S
D) R - S

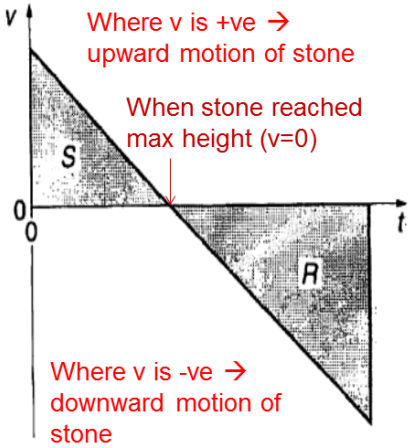
Solution:

Area under v-t graph gives CHANGE in displacement.



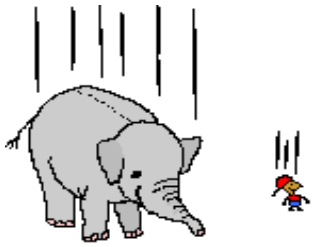
What is the height of the cliff-top above the sea? (N98/I/3)

A) R
B) S
C) R+ S
D) R - S



Quiz

Would a more massive object (e.g. elephant) fall faster than a less massive object (e.g. mouse) in a vacuum?



Solution:

All objects under free fall accelerates at 9.81 m s^{-2}. Hence both objects speed up from rest at the same rate (speed increases by 9.81 m s^{-1} per second).

MINI-TEST 2

A tennis ball is dropped from a height of 2.0 m above a hard level floor and it rebounds to a height of 1.5 m.

From the instant it is released to the point where it reaches maximum height after the first bounce,

(a) calculate the speed just after impact with the floor;
(b) calculate the total time taken and;
(c) sketch the velocity-time graph for this entire motion.

My solution:

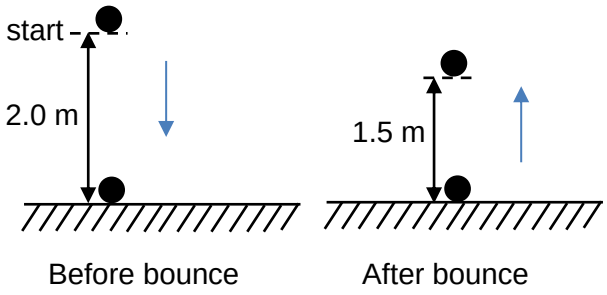
(a) Take upwards as positive

$v = 0$ m s^{-1}
$s = 1.5$ m
$a = -9.81$ m s^{-2}

Using $v^2 = u^2 + 2as$

$(0)^2 = (u)^2 + 2(-9.81)(1.5)$

$u = 5.42$ m s^{-1}



(b) Take upwards as positive

Before bounce:

$u = 0$ m s^{-1}
$s = -2.0$ m
$a = -9.81$ m s^{-2}

Using $s = ut + \frac{1}{2} a t^2$

$-2.0 = 0 + \frac{1}{2}(-9.81) t^2$

$t = 0.639$ s

After bounce:

$u = 5.42$ m s^{-1}, $v = 0$ (at highest point)
$s = 1.5$ m
$a = -9.81$ m s^{-2}

$v = u + at$

$0 = 5.42 + (-9.81)t$

$t = 0.552$ s

Total time = 0.639 + 0.552

= 1.19 s

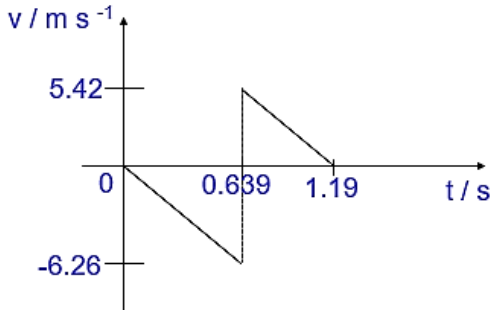
(c) Take **upwards as positive**.

Calculation of v just before the bounce:

Using $v^2 = u^2 + 2as$

$v^2 = 0 + 2(-9.81)(-2.0)$

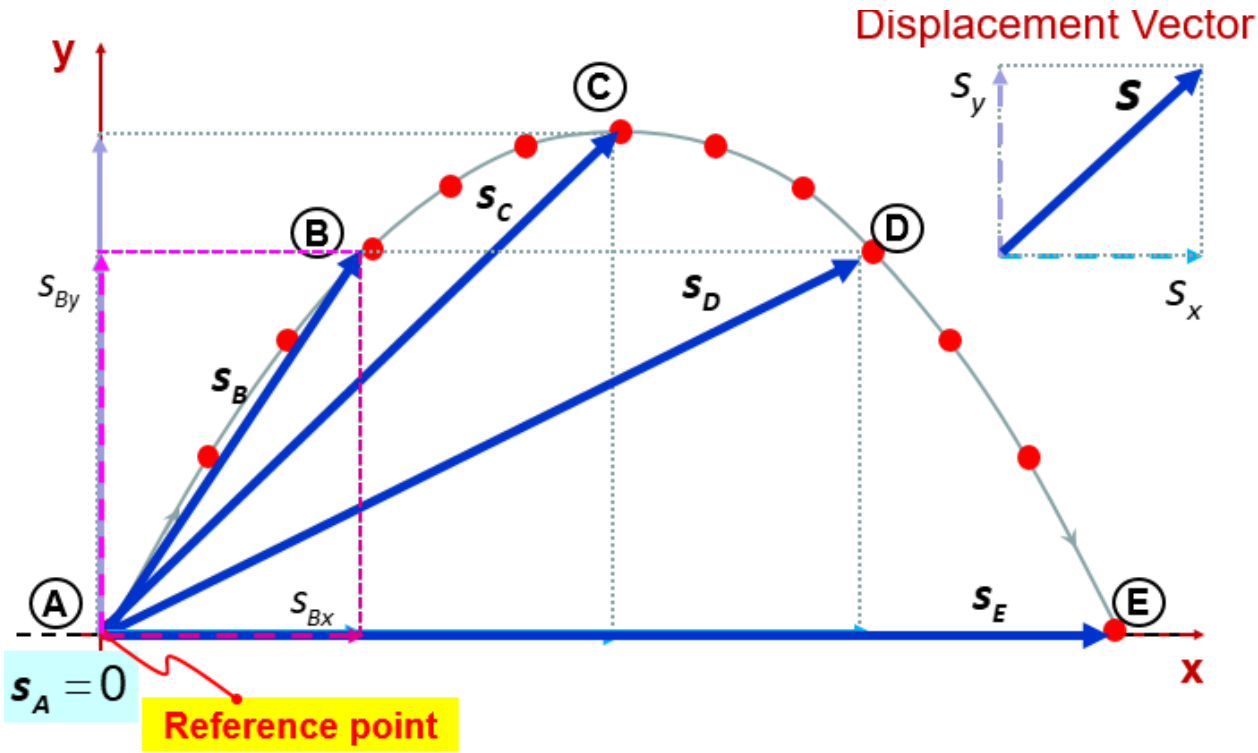
$v = -6.26 \text{ m s}^{-1}$



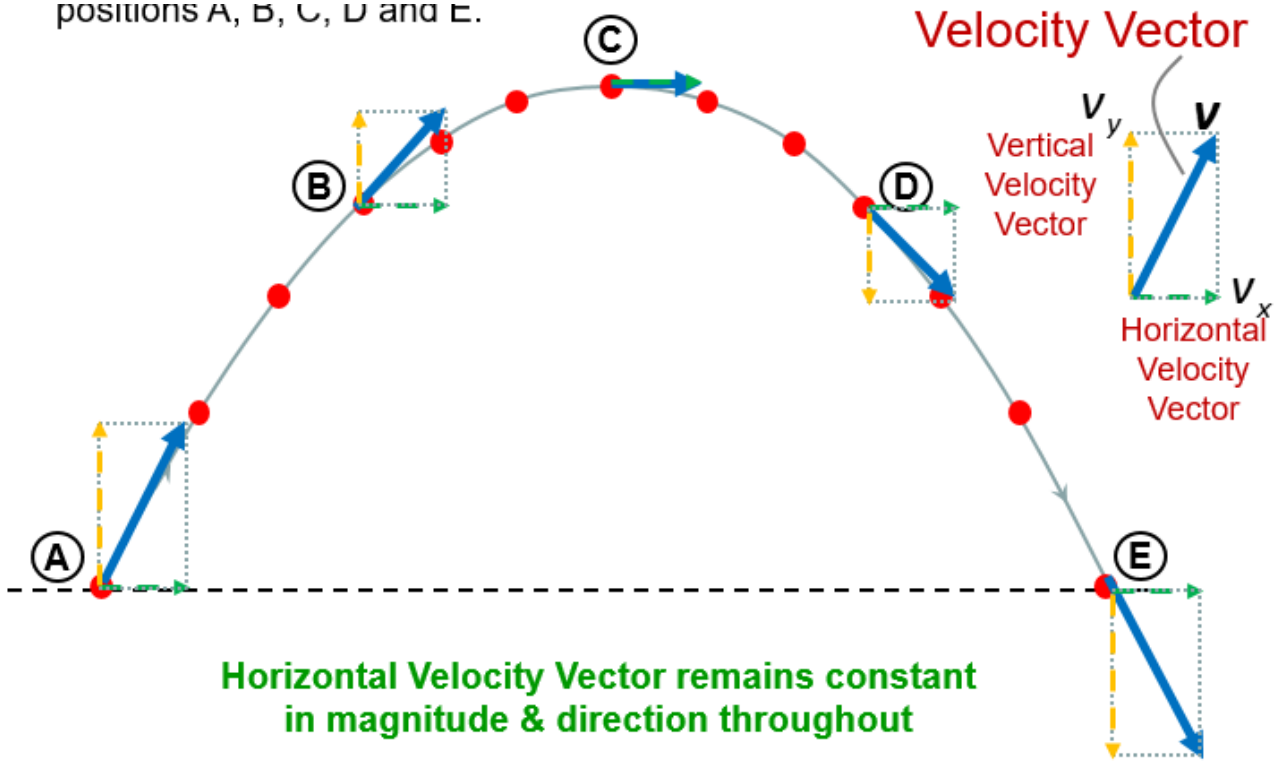
- Same sign convention is used for the graph.
- The gradient of the two segments are the same. And the areas bounded by the graph and the x-axis are not.

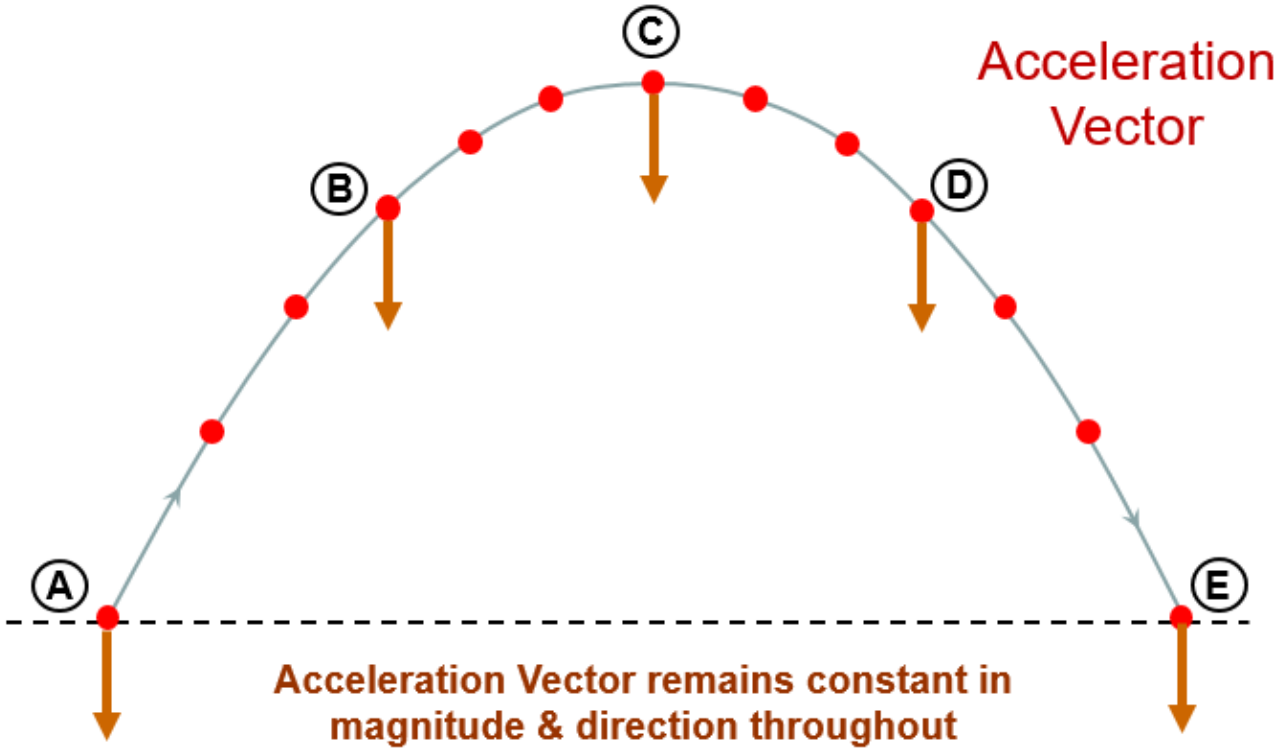
Quiz

For the scenario of a ball being projected in a parabolic path from point A, draw vectors to represent the displacement, velocity and acceleration at positions A, B, C, D and E.



positions A, B, C, D and E.

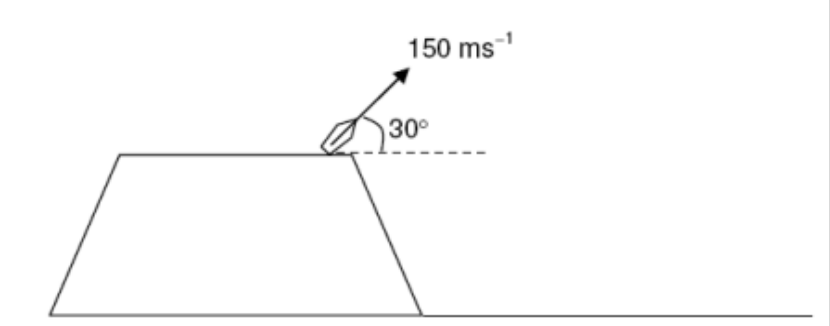




MINI-TEST 3

1. A grasshopper jumps at an angle of 30° to the horizontal with a take-off speed of 3.0 m s^{-1}.
   (a) What is the height of its jump?
   (b) How long is it above the ground?
   (c) What is the range of its jump?

2. A projectile is fired from the top of a cliff with a velocity of 150 m s^{-1} at 30° to the horizontal as shown in the diagram. If the time of flight is 22.5 s, what is the height of the cliff?



3. A tennis ball is thrown horizontally at 10 m s^{-1} from a height of 2.0 m above flat horizontal ground. How far away from the thrower does it first hit the ground?

My solutions:

1.

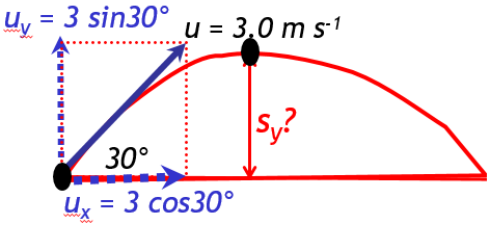
Step 2:

Define ref. pt. as the initial position of the body.

Choose up as +ve, right as +ve

(a) Height of jump?

Step 1:



Step 3 & 4:

Consider VERTICAL motion

$u_y = 3 \sin 30°$ m s^{-1}; $v_y = 0$;
$a_y = -9.81$ m s^{-2};
To find: s_y at max. ht. ($v_y = 0$)

Use $v_y^2 = u_y^2 + 2 a_y s_y$

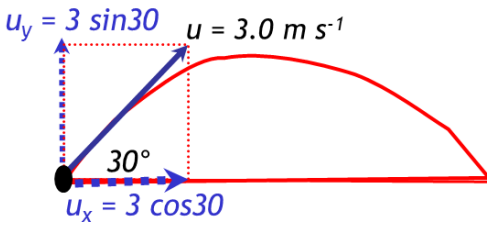
$0 = (3\sin 30°)^2 + 2(-9.81)(s_y)$
$s_y = 0.11$ m

Step 2:

Choose up as +ve, right as +ve

(b) Duration above ground?

Step 1:



Consider VERTICAL motion

Step 3 & 4:

$u_y = 3 \sin 30\ m\ s^{-1}$; $v_y = 0$;
$a_y = -9.81\ m\ s^{-2}$;
To find: t at max. height then multiply by 2

Method 2:
Find t when $s_y = 0$
Use $s_y = u_y t + \frac{1}{2} a_y t^2$

Step 5:

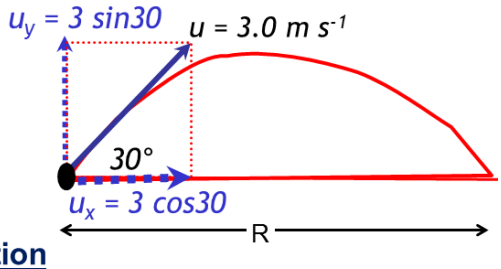
$v_y = u_y + a_y t$
$0 = (3\sin 30) + (-9.81)t$
$t = 0.153$ s (time to reach max height)
➔ Total time above the ground
$= 2 \times 0.153 = \underline{0.31\ s}$

Step 2:

Choose up as +ve, right as +ve

(c) Range of jump?

Step 1:



Consider HORIZONTAL motion

Step 3 & 4:

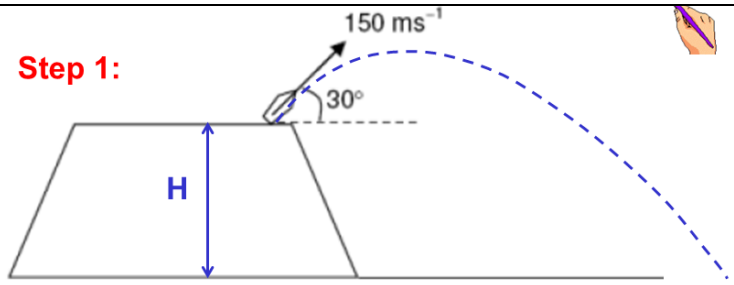
$u_x = 3 \cos 30\ m\ s^{-1}$; t = 0.306 s;
To find: s_x

Step 5:

$s_x = u_x t$ = range
$s_x = (3 \cos 30)\ (0.306)$
$= \underline{0.80\ m}$

2.

Step 1:



Step 2: Choose upward as +ve, right as +ve

Consider the VERTICAL motion

Step 3 & 4:

u_y = 150sin 30° ; a_y = -9.81 m s^{-2};
t = 22.5 s

To find: H, i.e. s_y when t = 22.5 s

Step 5:

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

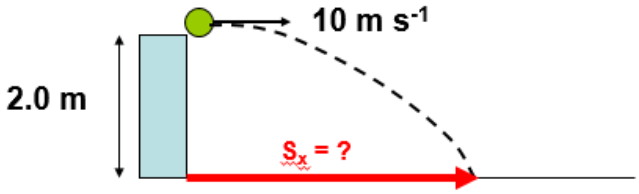
$$-H = (150 \sin 30^0)(22.5) + \frac{1}{2}(-9.81)(22.5)^2$$

$$-H = -796\ m$$

$$H = 796\ m$$

3.

Step 1:



Step 2:
Define ref. pt. as the initial position of the body.
Choose downward as +ve, right as +ve

Step 3 & 4:

Consider the HORIZONTAL motion

$s_x = u_x t$

To find: s_x

Known: u_x = 10 m s^{-1}; t = ?

Find using a vertical motion equation!

$$s_x = u_x t$$
$$= 10 \times 0.639$$
$$= 6.4\ m$$

Consider the VERTICAL motion

u_y = 0; a_y = +9.81 m s^{-2}; s_y = +2.0 m

To find: t when s_y = +2.0 m

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$2.0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.64\ s$$

Quiz

The diagram shows the parabolic path of a ball thrown in air. On the same diagram, draw the path of the ball if air resistance is **not** negligible.

Solution:

- ➔ *Knowledge that both horizontal & vertical velocities are affected. Horizontal velocity decreases to ZERO (if flight time is long enough).*
- ➔ *Asymmetric (horizontal distance travelled per unit time DECREASES due to horizontal deceleration), reaches maximum height earlier, lower maximum height, shorter range*