National Junior College
2016 – 2017 H2 Further Mathematics
Topic F8: Further Special Discrete Probability Distributions **Lecture Notes**

Key Questions to Answer:

A. The Poisson Distribution

1. What is a Poisson random variable?
2. Why is a Poisson random variable discrete?
3. What are the conditions for a random variable to be modelled by a Poisson distribution?
4. What is the probability distribution function of this distribution?
5. How do we calculate the mean and variance of a Poisson distribution?
6. How do we calculate probabilities for the sum of two or more independent Poisson random variables?
7. How is the binomial distribution related to the Poisson distribution?

B. The Geometric Distribution

1. What is a geometric random variable?
2. Why is a geometric random variable discrete?
3. What are the conditions for a random variable to be modelled by a geometric distribution?
4. What is the probability distribution function of this distribution?
5. How do we calculate the mean and variance of a geometric distribution?
6. What is the memoryless property of a geometric random variable?

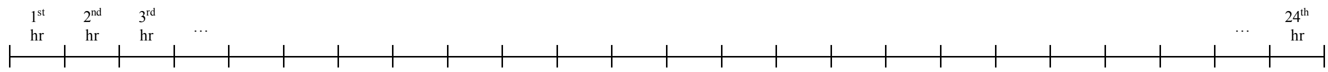
Previously, you have learnt a particular type of discrete probability distribution, called the binomial distribution. In this chapter, we will explore two other discrete probability distributions, called the *Poisson* and *geometric* distributions.

A. THE POISSON DISTRIBUTION

§1 Definition and Theory

Suppose you often receive WhatsApp messages and you are interested to know how likely you are to receive a certain number of WhatsApp messages in a certain day. Assuming that on average, you receive 9 WhatsApp messages each day, how do you find the probability that you receive, say, 2 WhatsApp messages in a particular day?

To start off, we can partition the day equally into 24 **disjoint** one-hour periods, as illustrated in the diagram below:



Assuming that you receive WhatsApp messages at a constant average rate throughout the day, the probability that you receive a WhatsApp message in any of the 24 one-hour periods is $\frac{9}{24} = 0.375$.

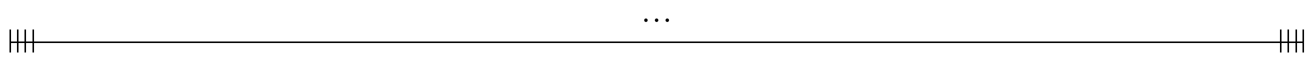
Let us now make the assumption that in <u>every one-hour period</u>, you either receive **no** WhatsApp message or only **one** WhatsApp message. *(Realistically, this is not a valid assumption to make – but let us leave this issue aside for now.*) We can then model the distribution for the number of WhatsApp messages that you receive in this particular day by a **binomial distribution** *approximately*, as follows:

$$X \sim \mathrm{B}(24, 0.375) \text{ approximately.}$$

Then, the required probability can be approximated as

$$\mathrm{P}\left(X = 2\right) \approx \binom{24}{2}(0.375)^2(1-0.375)^{24-2} = 0.0012540995.$$

Now to address the issue that you may potentially receive more than one message in the same one-hour period, we can improve the model, we can partition the day into finer intervals, say into 1440 one-minute periods, as illustrated below:



In this case, the probability that you receive a message in each one-minute period is $\frac{9}{1440} = \frac{1}{160}$, and therefore,

$$X \sim \mathrm{B}\left(1440, \frac{1}{160}\right) \text{ approximately.}$$

Therefore $\mathrm{P}\left(X = 2\right) \approx \binom{1440}{2}\left(\frac{1}{160}\right)^2\left(1-\frac{1}{160}\right)^{1440-2} = 0.0049168056.$

Notice now that the approximate probability value has changed significantly. To yield better approximations, we can further partition the day into *even finer* intervals, say one-second intervals, which yield the following approximate distribution for *X*:

$$X \sim \mathrm{B}\left(86400, \frac{1}{9600}\right) \text{ approximately.}$$

Therefore $\mathrm{P}(X=2) \approx \binom{86400}{2}\left(\frac{1}{9600}\right)^2\left(1-\frac{1}{9600}\right)^{86400-2} = 0.0049967376.$

Notice that the approximate probability value starts to change *less* significantly. If we continue this process and further partition the day into even finer intervals (of equal duration), we obtain the following approximate probability values:

n	$p\left(=\frac{9}{n}\right)$	approximate value for $\mathrm{P}(X=x)$
100000	0.00009	0.004996923
10000000	0.0000009	0.004998085
600000000	0.000000015	0.004998097

Notice that the approximate values of $\mathrm{P}(X=2)$ *converges* to a limit as $n \to \infty$, which is given by

$$\begin{aligned}
\lim_{n\to\infty}\left[\binom{n}{2}\left(\frac{9}{n}\right)^2\left(1-\frac{9}{n}\right)^{n-2}\right] &= \lim_{n\to\infty}\left[\frac{n!}{2!(n-2)!}\left(\frac{9}{n}\right)^2\left(1-\frac{9}{n}\right)^{-2}\left(1+\frac{(-9)}{n}\right)^n\right] \\
&= \lim_{n\to\infty}\left[\frac{n!}{2!(n-2)!}\left(\frac{9}{n}\right)^2\left(\frac{n-9}{n}\right)^{-2}\left(1+\frac{(-9)}{n}\right)^n\right] \\
&= \lim_{n\to\infty}\left[\frac{n(n-1)}{2!}\left(\frac{9}{n-9}\right)^2\left(1+\frac{(-9)}{n}\right)^n\right] \\
&= \lim_{n\to\infty}\left[\frac{n(n-1)}{(n-9)^2}\left(\frac{9^2}{2!}\right)\left(1+\frac{(-9)}{n}\right)^n\right] \\
&= \frac{9^2}{2!}\lim_{n\to\infty}\frac{n(n-1)}{(n-9)^2}\times\lim_{n\to\infty}\left[\left(1+\frac{(-9)}{n}\right)^n\right] \\
&= \frac{9^2}{2!}\cdot 1\cdot \mathrm{e}^{-2} = \frac{\mathrm{e}^{-2}9^2}{2!} = 0.0049980971
\end{aligned}$$

[For the above calculations, we make use of the result that $\lim_{n\to\infty}\left[\left(1+\frac{x}{n}\right)^n\right] = \mathrm{e}^x$ for any $x \in \mathbb{R}$.]

In general, one can prove the following result.

Result 1.1 If λ is a **positive** constant, then,

$$\lim_{n\to\infty}\left[\binom{n}{x}p^x(1-p)^{n-x}\right] = \frac{\mathrm{e}^{-\lambda}\lambda^x}{x!},$$

where $p = \frac{\lambda}{n}$.

In general, random variables that count the number of times a particular event occur throughout a given region of time or space, such as the scenario illustrated above, i.e. the number of WhatsApp messages you receive in a day, can usually be modelled as having a probability distribution function whose rule can be expressed in the form as shown in the right-hand side of Result 1.1, subject to certain conditions. Random variables having such a probability distribution function are called **Poisson random variables**, which is formally defined below.

Definition 1.1 (Poisson Random Variable)

A discrete random variable X which takes values 0, 1, 2, 3, ..., such that

$$\mathrm{P}\left(X=x\right)=\frac{\mathrm{e}^{-\lambda}\lambda^{x}}{x!},\ \textbf{(in MF26)}$$

where λ is a **positive** constant, is said to be a *Poisson random variable* or to have a *Poisson distribution* with parameter λ.

Notation: $X \sim \mathrm{Po}(\lambda)$.

Conditions for using a Poisson Model

A Poisson random variable must satisfy the following conditions:

No.	Condition	Description
1	**Randomness**	Events occur randomly throughout the region of time or space.
2	**Constant Mean Rate**	Events occur at a constant mean rate throughout the region of time or space, i.e. the average number of events occurring per unit time or space is a constant.
3	**Independence**	Events occur independently of one another throughout the region of time or space.

Examples of Poisson Random Variables

- the number of particles emitted in a minute by a radioactive source,
- the number of telephone calls within a fixed time interval at a switchboard,
- the number of typographical errors in a randomly chosen page of a book,
- the number of car accidents occurring on a particular stretch of road in a particular day.

Result 1.2 If λ is a **positive** constant, the function

$$\mathrm{f}(x)=\frac{\mathrm{e}^{-\lambda}\lambda^{x}}{x!},\ x=0,1,2,3,\ldots$$

is a probability distribution function.

Proof: We need to show that

(1) For all $x=0,1,2,3,\ldots,\ 0\le \mathrm{f}(x)\le 1,$ and

(2) $\sum_{x=0}^{\infty}\mathrm{f}(x)=1.$

Note that since λ is positive, $\lambda^{x}>0$ for all $x=0,1,2,3,\ldots.$ Also, $\mathrm{e}^{-\lambda}$ and $x!$ are also always positive by definition. Hence $\mathrm{f}(x)>0$ for all $x=0,1,2,3,\ldots.$

It now suffices to prove that $\sum_{x=0}^{\infty}\mathrm{f}(x)=1$ to show that $\mathrm{f}(x)$ satisfies all criteria of a probability distribution function, since if this is true, then for all $x=0,1,2,3,\ldots,\ \mathrm{f}(x)\le\sum_{x=0}^{\infty}\mathrm{f}(x)=1$ since $\mathrm{f}(x)>0$ for all $x=0,1,2,3,\ldots.$

Mean and Variance

Result 1.3 (Mean and Variance of the Poisson Distribution) If $X \sim \text{Po}(\lambda)$, then

$$\text{E}(X) = \lambda \text{ and } \text{Var}(X) = \lambda \text{. } \textbf{(in MF26)}$$

Proof:

$$\text{E}(X) = \sum_{x=0}^{\infty} x\text{P}(X = x)$$

To find Var(X), we first find $\text{E}\left[X(X-1)\right]$.

$$\text{E}\left[X(X-1)\right] = \sum_{x=0}^{\infty} x(x-1)\text{P}(X = x)$$

Hence $\text{E}\left(X^2\right) = \text{E}\left(X^2 - X + X\right)$

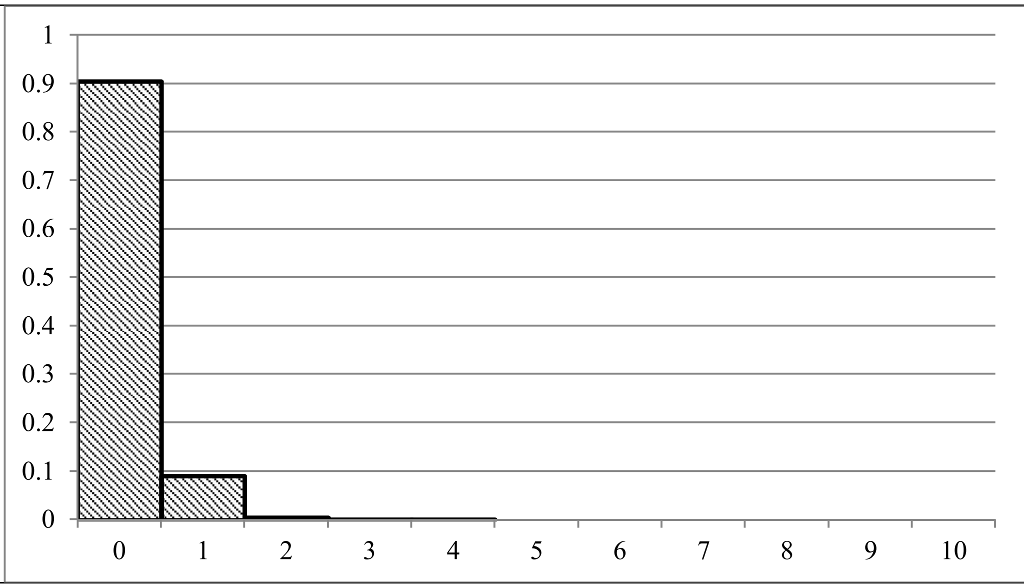
$$= \text{E}\left[X(X-1)\right] + \text{E}(X)$$

and $\text{Var}(X) = \text{E}\left(X^2\right) - \left[\text{E}(X)\right]^2$

Probability Histograms for the Poisson Distribution

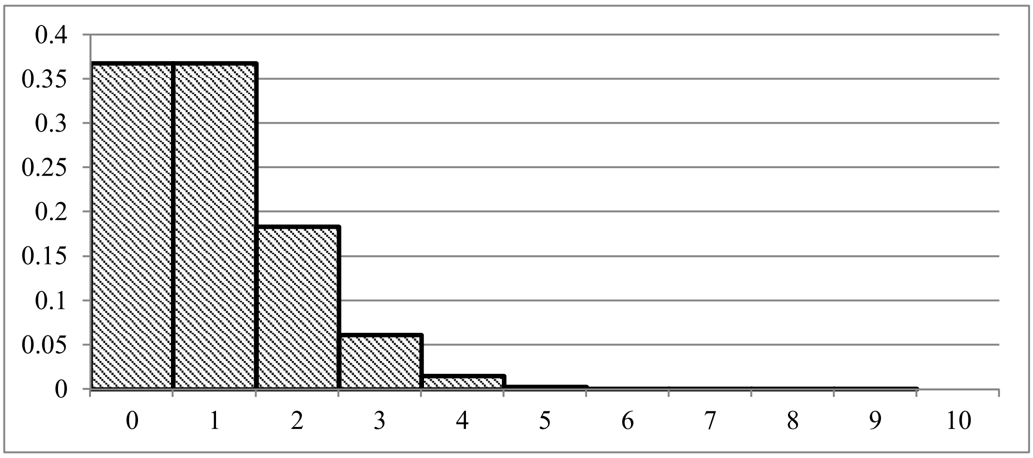
$X \sim \text{Po}(0.1)$

x	$\text{P}(X=x)$
0	0.904837
1	0.090484
2	0.004524
3	0.000151
4	3.77E-06
5	7.54E-08
6	1.26E-09
7	1.8E-11
8	2.24E-13
9	2.49E-15
10	2.49E-17



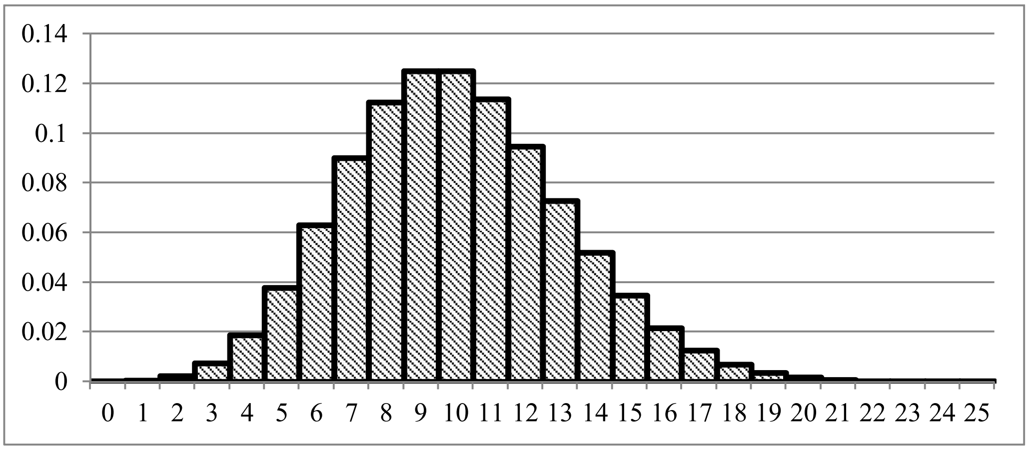
$X \sim \text{Po}(1)$

x	$\text{P}(X=x)$
0	0.367879
1	0.367879
2	0.18394
3	0.061313
4	0.015328
5	0.003066
6	0.000511
7	7.3E-05
8	9.12E-06
9	1.01E-06
10	1.01E-07



$X \sim \text{Po}(10)$

x	$\text{P}(X=x)$
0	4.54E-05
1	0.000454
2	0.00227
3	0.007567
4	0.018917
…	…
21	0.000889
22	0.000404
…	…



Note that

(a) when λ is small, the histogram for the Poisson distribution is skewed to the right,

(b) for large values of λ, the histogram for the Poisson distribution is symmetrical about its mean.

Example 1.1
An average of 60 customers enter a particular store every hour. Explain, *in context*, why the number of customers entering the store in an hour may be well-modelled by a Poisson distribution. Hence, find the probability that no one enters the store during a particular five-minute period.

Solution:

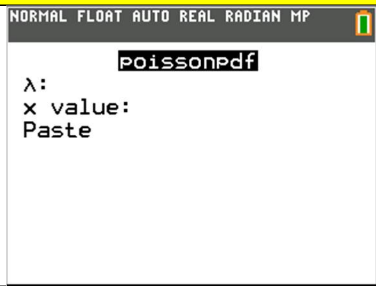
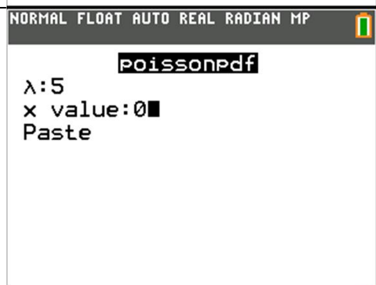
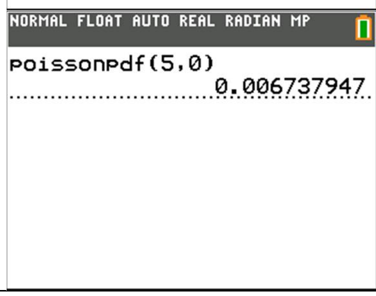
Let X denote the number of customers entering the store in a particular **five-minute** period.

Since we expect that the customers enter the store at a constant mean rate, $\mathrm{E}(X)=\frac{5}{60}\times 60=5.$ So

$$X \sim \mathrm{Po}(5).$$

Therefore $\mathrm{P}(X=0)=\dfrac{\mathrm{e}^{-5}5^{0}}{0!}=\mathrm{e}^{-5}=0.0067379=0.00674$ (to 3 s.f.)

Alternatively, we can use the graphing calculator to obtain the answer, as illustrated below.

Instructions	Screenshot
Press 2nd VARS, select 'C: poissonpdf(' and press ENTER.	
Key in **5, 0.** At 'Paste', press ENTER twice and you will obtain the answer.	 

Example 1.2
An entomologist sets a moth-trap and counts the number of 'Blue Moon' moths caught in the trap. Over a particular month, it is found that the mean number of 'Blue Moon' moths caught per evening is 3.5.

Assuming that the number of moths caught during this month follows a Poisson distribution, find the probability that
(i) two 'Blue Moon' moths are caught in a randomly chosen evening,
(ii) not less than five 'Blue Moon' moths are caught in a randomly chosen evening,
(iii) at least 10 'Blue Moon' moths are caught in two randomly chosen evenings.

Explain why a Poisson model would probably not be valid if applied to a time period of a year.
[N2004/II/28(part)(adapted)]

Solution:
Let X denote the number of 'Blue Moon' moths caught in a randomly chosen evening. Then

$$X \sim \mathrm{Po}(3.5).$$

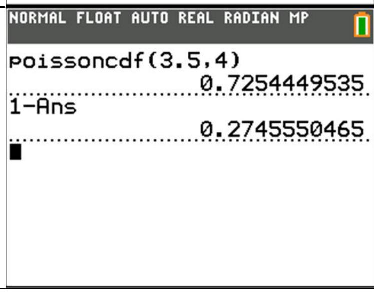
(i)

(ii) $\mathrm{P}(X \geq 5) = 1 - \mathrm{P}(X \leq 4)$

$$= 1 - \left[\mathrm{P}(X=0)+\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=3)+\mathrm{P}(X=4)\right]$$

$$= 1 - \mathrm{e}^{-3.5}\left(1 + 3.5 + \frac{3.5^2}{2!} + \frac{3.5^3}{3!} + \frac{3.5^4}{4!}\right) = 0.27456 = 0.274 \text{ (to 3 s.f.)}$$

Alternatively, we can use GC to obtain the answer for part **(ii)**, as illustrated below.

Instructions	Screenshot
Press **2nd** **VARS**, select 'D: poissoncdf(' and press **ENTER**. *The 'poissoncdf' function calculates probabilities of the form* $\mathrm{P}(X \leq x)$.	
Evaluate P($X \leq 4$) by keying in **3.5, 4**. At 'Paste', press **ENTER** twice.	
To find P($X \geq 5$), subtract the answer obtained above from 1.	

(iii) Let Y denote the number of 'Blue Moon' Moths caught in two randomly chosen evenings respectively. Then

$$Y \sim \text{Po}(2\times 3.5) \Rightarrow Y \sim \text{Po}(7).$$

$P\left(Y \le 10\right) = 0.90148 = 0.901$ (to 3 s.f.)

§2 The Additive Property of Independent Poisson Distributions

Result 2.1 (Additive Property of Independent Poisson Distributions)

If X and Y are independent Poisson random variables with mean λ and μ respectively, then

$$X + Y \sim \text{Po}(\lambda + \mu).$$

Proof: Suppose $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$, and X and Y are independent. Then

$$\begin{aligned}\text{P}(X+Y=w) &= \sum_{x=0}^{w} \text{P}(X = x \textbf{ and } Y = w-x) \\ &= \sum_{x=0}^{w} \text{P}(X=x)\times\text{P}(Y=w-x) \text{ (since } X \text{ \& } Y \text{ are independent)}\end{aligned}$$

Therefore, $X+Y \sim \text{Po}(\lambda+\mu)$.

Example 2.1

An office telephone switchboard handles both internal and external calls. The number of internal calls and external calls that arrive at the switchboard in any ten-minute period have the independent distributions Po(3) and Po(2) respectively.

(a) Find the probability that in a given ten-minute period,

(i) a total of five calls arrive at the switchboard.

(ii) exactly four internal calls and one external call arrive at the switchboard.

(b) Find the probability that in three disjoint five-minute periods, exactly one call arrives at the switchboard in exactly two of the periods and more than three calls arrive at the switchboard in the other period.

(c) It is given that there is a probability of more than 0.95 that there is at least one external call arriving at the switchboard in a period of n whole minutes. Write down an inequality in terms of n, and hence find the least value of n.

Solution:

(a) Let X and Y be the random variables for the number of internal and external calls arriving in a ten–minute period respectively. Then

$$X \sim \text{Po}(3) \text{ and } Y \sim \text{Po}(2).$$

Since X and Y are independent,

$$X + Y \sim \text{Po}(5).$$

Therefore,

(i)

(ii) $\text{P}(X = 4 \textbf{ and } Y = 1) = \text{P}(X = 4) \times \text{P}(Y = 1)$ (since X and Y are independent)

$$= 0.045481 = 0.0455 \text{ (to 3 s.f.)}$$

(b) Let W be the random variable for the total number of calls that arrive at the switchboard in a five-minute period. Then

$$W \sim \text{Po}(2.5).$$

(b) Let V be the random variable for the number of external calls that arrive at the switchboard in a period of n minutes. Then

$$V \sim \text{Po}(0.2n).$$

Example 2.2
A water tank containing 10^4 ml of water which is initially free of a certain kind of micro-organism. 1200 such micro-organisms are placed into the water and the mixture is stirred to distribute the organisms randomly throughout the water. Five test-tubes are then each filled with 10 ml of the mixture. Find the probability that

(i) a particular test-tube contains two micro-organisms,

(ii) exactly three of the five test-tubes contain two micro-organisms each,

(iii) the five test-tubes contain a total of four micro-organisms.

[You may assume that the micro-organisms have negligible volume.]

Solution:

(i) Let X denote the number of micro-organisms in a test-tube containing 10ml of the mixture. Then

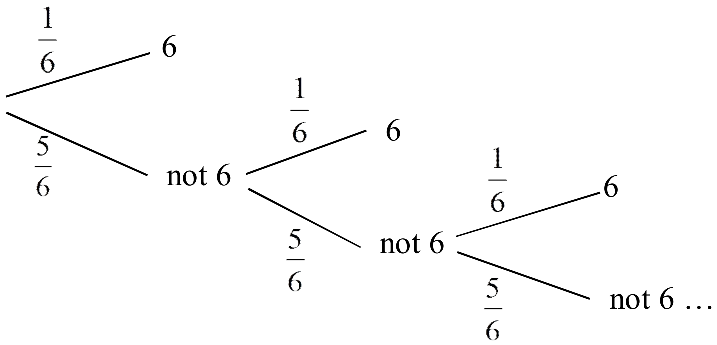
$$X \sim \mathrm{Po}\left(\frac{1200}{10^4}\times 10\right) \Rightarrow X \sim \mathrm{Po}(1.2).$$

$\mathrm{P}(X=2)=0.21686=0.217$ (to 3 s.f.)

B. THE GEOMETRIC DISTRIBUTION

§1 Definition and Theory

Suppose you roll a fair die until you get a 6. The scenario can be illustrated by a tree diagram as shown below:



The probabilities of throwing the die x times, for $x = 1, 2, 3,\ldots$ are listed in the table below.

x	1	2	3	…
$\mathrm{P}(X = x)$	$\frac{1}{6}$	$\left(\frac{5}{6}\right)\frac{1}{6}$	$\left(\frac{5}{6}\right)^2\frac{1}{6}$	…

In general,

$$\mathrm{P}(X = x) = \frac{1}{6}\left(\frac{5}{6}\right)^{x-1}$$

Random variables that count the number of trials of a particular experiment that need to be conducted before a particular outcome ("success") occurs, such as the one depicted in the example above, are known as **geometric random variables**, which are formally defined below.

Definition 1.1 (Geometric Random Variable)
A discrete random variable X which takes values 0, 1, 2, 3, ..., such that

$$\mathrm{P}(X = x) = p(1-p)^{x-1},$$

where p is a constant such that $0 < p < 1$, is said to be a *geometric random variable* or to have a *geometric distribution* with parameter p.

<u>Notation</u>: $X \sim \mathrm{Geo}(p)$.

<u>Conditions for using a Geometric Model</u>
A geometric random variable must satisfy the following conditions, which are similar to those for the binomial random variable:

No.	Condition	Description
1	**Possible Outcomes of each Trial**	There are exactly two possible outcomes for each trial, success and failure.
2	**Constant Probability of Success**	The probability of success for each trial, p, must be a constant.
3	**Independence**	The events of failures (or success) occur independently across the trials.

Examples of Geometric Random Variables

- the number of coin tosses made up to and including the first time a heads appears,
- the number of people surveyed on a street up to and including the first person who has a particular blood type, say A,
- the number of people walking past until a flyer distributer up to and including the first person who accepts a flyer from him/her

Result 1.2 If λ is a **positive** constant, the function

$$\mathrm{f}(x) = p(1-p)^{x-1}, x = 1, 2, 3, \ldots,$$

where p is a constant such that $0 < p < 1$, is a probability distribution function.

Proof: We need to show that

(1) For all $x = 1, 2, 3, \ldots,\ 0 \leq \mathrm{f}(x) \leq 1,$ and

(2) $\displaystyle\sum_{x=1}^{\infty} \mathrm{f}(x) = 1.$

(1) Note that p is a constant such that $0 < p < 1$, $0 < 1 - p < 1$ as well.

Therefore, $0 < p(1-p)^x < 1$ for all $x = 1, 2, 3, \ldots.$

(2) $\displaystyle\sum_{x=1}^{\infty} \mathrm{f}(x) = \sum_{x=1}^{\infty} p(1-p)^{x-1}$

Mean and Variance

Result 1.3 (Mean and Variance of the Poisson Distribution) If $X \sim \text{Geo}(p)$, then

$$\mathrm{E}\left(X\right)=\frac{1}{p} \text{ and } \mathrm{Var}\left(X\right)=\frac{1-p}{p^2}.$$ **(in MF26)**

Proof (for mean):

$$\mathrm{E}(X)=\sum_{x=1}^{\infty} x\mathrm{P}(X=x)$$

Proof (for variance):

Differentiating both sides of the identity $(1-x)^{-2}=\sum_{k=1}^{\infty}kx^{k-1}$ with respect to x (for $|x|<1$),

Therefore,

$$\mathrm{E}\left[X\left(X-1\right)\right]=\sum_{x=1}^{\infty}x(x-1)\mathrm{P}(X=x)$$

Result 1.4 If $X \sim \text{Geo}(p)$, then $\text{P}(X > k) = (1-p)^k$.

Proof:

$$\text{P}(X > k) \;=\; \sum_{x=k+1}^{\infty} \text{P}(X = x)$$

Alternatively, note that the event that success occurs only after the k^{th} trial is the event that failures occur in all of the first k trials. Therefore, $\text{P}(X > k) \;= (1-p)^k$.

Result 1.5 (Memoryless Property) If $X \sim \text{Geo}(p)$, then

$$\text{P}(X > m+n \mid X > n) = \text{P}(X > m),$$

for any non-negative integers n and m.

Proof:

$$\text{P}\left(X > m+n \middle| X > n\right) = \frac{\text{P}\left(X > m+n \cap X > n\right)}{\text{P}\left(X > n\right)}$$

In other words, if it is already known that failures have occurred in all of the first n trials, then the probability that failures occur in all of the first m + n trials is the equal to the probability that failures occur in all of the next m trials, by virtue of the fact that the outcomes of the next m trials are independent of the outcomes in the first n trials. This is called the *memoryless property* of the geometric distribution.

Example 1.1

Single items from the output of a machine are extracted and tested. It is known that every one out of 20 items is faulty. Find

(a) the probability that the first faulty item found is the 8^{th} one selected,

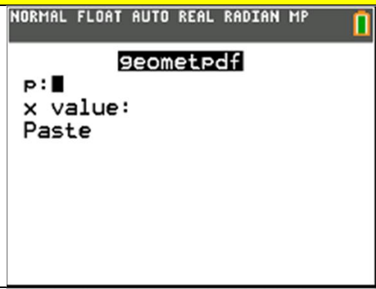
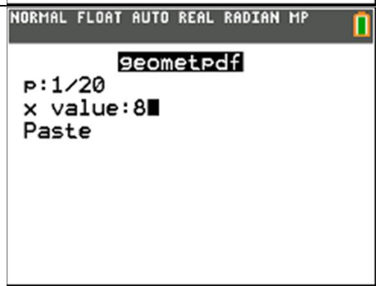
(b) the probability that the first faulty item found comes before the 8^{th} one selected,

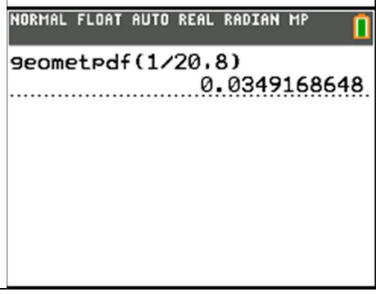
(c) the expected number of items extracted before a faulty one is found.

Solution:
Let X be the random variable representing the number of items selected until the first faulty item is found. Then $X \sim \text{Geo}\left(\frac{1}{20}\right)$.

(a) $\mathrm{P}\left(X=8\right)=\left(\frac{19}{20}\right)^7\left(\frac{1}{20}\right)=0.034917=0.349$ (to 3 s.f.)

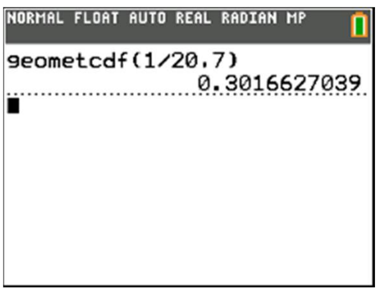
Alternatively, we can use the graphing calculator to obtain the answer, as illustrated below.

<table>
<tr><th>Instructions</th><th>Screenshot</th></tr>
<tr><td>Press 2nd VARS, select 'E: geometpdf(' and press ENTER.</td><td>
</td></tr>
<tr><td>Key in 1/20, 8. At 'Paste', press ENTER twice and you will obtain the answer.</td><td>


</td></tr>
</table>

(b)
$$\begin{aligned}\mathrm{P}\left(X<8\right)&=1-\mathrm{P}(X\geq 8)\\&=1-\mathrm{P}(X>7)\\&=1-\left(\frac{19}{20}\right)^7\\&=0.30166=0.302 \text{ (to 3 s.f.)}\end{aligned}$$

Alternatively, the command "F: geometcdf" in the "distr" menu of the GC can be used to calculate the probability, as shown in the screenshot below:



$$\begin{aligned}P(X<8)&=P(X\le 7)\\&=0.30166\\&=0.302 \text{ (to 3 s.f.)}\end{aligned}$$

(c) $E(X)=\dfrac{1}{\frac{1}{20}}=20$

Example 1.2

Benson decides to make a telephone call to each of his friends one evening and to keep calling different friends until one of his friends successfully picks up his call. On average, the probability that any of his calls is successful is $\frac{4}{5}$.

(a) State two assumptions required for the number of phone calls Benson makes to be well-modelled by a geometric distribution, and explain why they may not be valid, in context.

Assume that the assumptions you have stated in part **(a)** now hold.

(b) What is the probability that Benson has to call
  (i) more than three of his friends,
  (ii) more than five of his friends, given that he has already called two of his friends?

(c) What is the expected number of phone calls that Benson makes the most likely number of phone calls that Benson makes?

Solution:

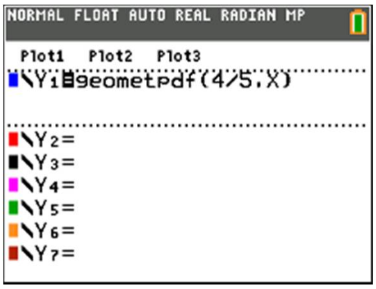
(a)

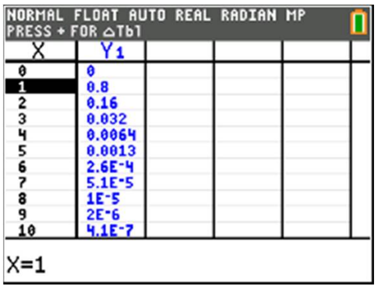
(b) Let X denote the number of calls Benson makes. Then $X \sim \text{Geo}\left(\frac{4}{5}\right)$. So

(i) $P(X>3)=\left(\frac{1}{5}\right)^3=\frac{1}{125}$

(ii)

(c) $\mathrm{E}\left(X\right)=\dfrac{1}{\frac{4}{5}}=\dfrac{5}{4}$





$\mathrm{P}(X=0)=0<0.8$
$\mathrm{P}(X=1)=0.8$
$\mathrm{P}(X=2)=0.16<0.8$

Therefore, the most likely number of calls that Benson makes is 1.