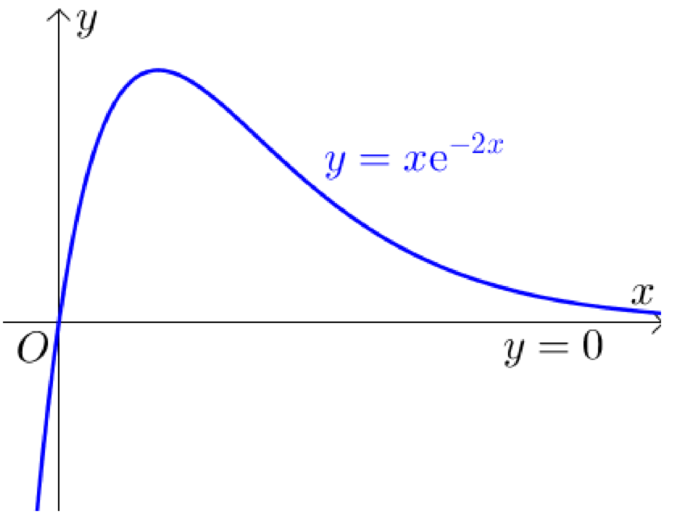
National Junior College
2016 – 2017 H2 Mathematics
Applications of Integration (Definite Integrals) **Assignment 1 Solutions**

1(i)



[B1] Shape (with curve passing through origin).

[B1] Equation of asymptote $y = 0$.

1(ii)

Observe from (i) that as $x \to \infty$,

$$xe^{-2x} \to 0.$$

Using integration by parts,

$$\int_1^\infty xe^{-2x}\,dx = \left[-\frac{x}{2}e^{-2x}\right]_1^\infty - \int_1^\infty -\frac{1}{2}e^{-2x}\,dx$$

$$= \left[0-\left(-\frac{1}{2}e^{-2}\right)\right] - \left[\frac{1}{4}e^{-2x}\right]_1^\infty$$

$$= \frac{1}{2}e^{-2} + \frac{1}{4}e^{-2}$$

$$= \frac{3}{4}e^{-2}.$$

Therefore $a = 0,\ b = \frac{3}{4}$.

[B1] $xe^{-2x} \to 0$.

[M1] Attempt to use integration by parts.

[A1] Correct choice of part of integrand to integrate and differentiate.

[A2,1,0] $a = 0,\ b = \frac{3}{4}$.

2

$$\int_a^{2\sqrt{3}} \frac{1}{4+x^2}\,dx = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_a^{2\sqrt{3}} = \left[\sin^{-1}x\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{\pi}{6} - \frac{1}{2}\tan^{-1}\frac{a}{2} = \frac{\pi}{3} - \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1}\frac{a}{2} = 0$$

$$\Rightarrow a = 0.$$

[B1] $\frac{1}{2}\tan^{-1}\frac{x}{2}$.

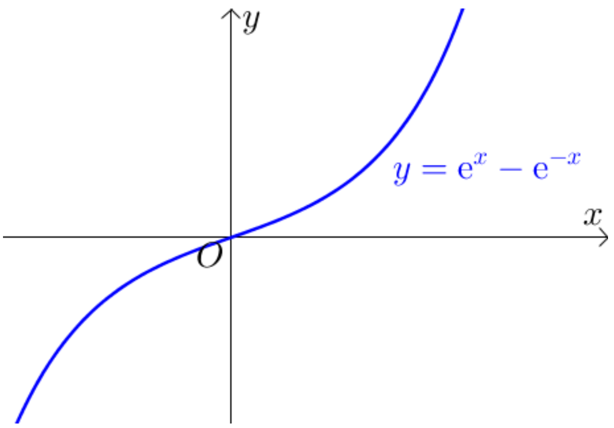
[B1] $\sin^{-1}x$.

[M1] Substitution.

[A1] $a = 0$.

3	$x=\sec^2\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta}=2\sec^2\theta\tan\theta.$	
	$\displaystyle\int_2^4 \frac{1}{x^2\sqrt{x-1}}\,\mathrm{d}x=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2\sec^2\theta\tan\theta}{\sec^4\theta\sqrt{\sec^2\theta-1}}\,\mathrm{d}\theta$	[M1] Attempt to substitute.
	$\displaystyle=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2}{\sec^2\theta}\,\mathrm{d}\theta$	[A1] $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2}{\sec^2\theta}\,\mathrm{d}\theta$
	$\displaystyle=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}2\cos^2\theta\,\mathrm{d}\theta$	
	$\displaystyle=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\cos 2\theta+1\,\mathrm{d}\theta$	[DM1] Integrate $\cos^2\theta$.
	$\displaystyle=\left[\frac{\sin 2\theta}{2}+\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$	
	$\displaystyle=\left(\frac{\sqrt{3}}{4}+\frac{\pi}{3}\right)-\left(\frac{1}{2}+\frac{\pi}{4}\right)$	
	$\displaystyle=\frac{\pi}{12}+\frac{\sqrt{3}-2}{4}.$	[A1] $\dfrac{\pi}{12}+\dfrac{\sqrt{3}-2}{4}.$

4(i)



Therefore $x>0$.

Alternatively, $\mathrm{e}^x-\mathrm{e}^{-x}>0$

$\Rightarrow\ \left(\mathrm{e}^x\right)^2-1>0$

$\Rightarrow\ \left(\mathrm{e}^x-1\right)\left(\mathrm{e}^x+1\right)>0$

$\Rightarrow\ \mathrm{e}^x-1>0 \qquad \left(\because \mathrm{e}^x+1 \text{ is always +ve}\right)$

$\Rightarrow\ \mathrm{e}^x>1$

$\Rightarrow\ x>\ln x=0.$

[M1] Graphical method or express LHS as quadratic: $\left(\mathrm{e}^x\right)^2-1.$

[A1] $x>0.$

4(ii)	$\int_{-4}^{3} \left\|e^x - e^{-x}\right\| dx$	
	$= \int_{-4}^{0} -\left(e^x - e^{-x}\right) dx + \int_{0}^{3} \left(e^x - e^{-x}\right) dx$	[M1] Split integrand.
	$= \left[-e^x - e^{-x}\right]_{-4}^{0} + \left[e^x + e^{-x}\right]_{0}^{3}$	[A1]
	$= \left(-2 + e^{-4} + e^{4}\right) + \left(e^{3} + e^{-3} - 2\right)$	[A1]
	$= 70.75$ (to 2 d.p.).	