National Junior College
2016 – 2017 H2 Mathematics
Revision: Trigonometry **Practice Questions**

Basic Mastery Questions

Basic Trigonometric Functions

1. By using a right-angled triangle, show that

   (a) $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$, (b) $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$,

   (c) $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$.

   Remark: Even though we have only justified the above results for an acute angle θ, in fact these relationships hold for ***all*** values of θ.

2. It is given that $A=120°$. Without using a calculator, write down the smallest positive integer n such that each of the following trigonometric expression yields a positive value. Justify your answer clearly.

   (a) $\sin(nA)$, (b) $\tan(nA)$,

   (c) $\cos(nA)$.

Relationship between Basic Trigonometric Functions and their inverses

3. Find the exact value of $\cos\left(\tan^{-1}1\right)$.

4. It is given that $\theta=\tan^{-1}x$, where x is positive.

   (a) State the range of possible values of θ.

   (b) Find the following in terms of x:

   (i) $\cot\theta$, (ii) $\sec\theta$,

   (iii) $\operatorname{cosec}\theta$.

Use of Formulae

5. It is given that $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$. Evaluate $\tan(A+B)$ and $\tan(2A)$ without the use of a calculator.

6. Express $\sin x+\sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$ are constants whose exact values are to be determined.

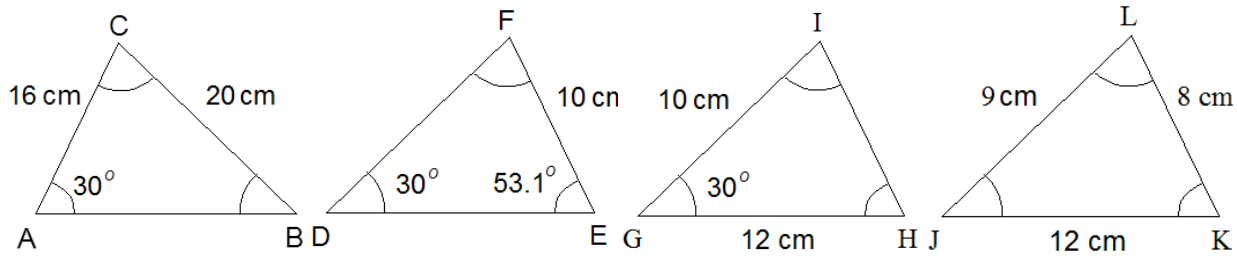
Trigonometric Equations

7. Write down the basic angle for $\cos x=\frac{1}{2}$. Hence solve $\cos x=\frac{1}{2}$, where $0\le x\le 2\pi$.

8. Write down the basic angle for $\tan x=-\frac{1}{\sqrt{3}}$. Hence solve $\tan x=-\frac{1}{\sqrt{3}}$, where $0\le x\le 2\pi$.

9. Solve $\sin\left(2x+\frac{\pi}{4}\right)=0$, where $0\le x\le 2\pi$.

Sine and Cosine Rule



10. The diagrams above (not drawn to scale) show four triangles. Find the possible value(s) of
    (a) $\angle ABC$,
    (b) DF,
    (c) HI,
    (d) $\angle JLK$.

Intermediate Level Questions

Compound Angle Formulae

1. Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, prove that one possible value of $\cos(A+B)$ is $\frac{33}{65}$, and find all the other possible values. **[J80/P1/4]**

Proving Trigonometric Identities

2. Prove the following identities:
    (a) $\dfrac{\sin A}{1-\cos 2A} = \dfrac{1}{2}\operatorname{cosec} A$,
    (b) $\sqrt{2+2\cos x} = 2\cos\dfrac{x}{2}$,
    (c) $\cos^4\theta - \sin^4\theta = \cos 2\theta$,
    (d) $\dfrac{\cos x + \sin x}{\cos x - \sin x} = \tan\left(x + \dfrac{\pi}{4}\right)$,
    (e) $\dfrac{\sin 5\theta + \sin\theta}{\cos 5\theta - \cos\theta} = -\cot 2\theta$,
    (f) $\sin\left(3x + \dfrac{\pi}{4}\right)\cos\left(3x - \dfrac{\pi}{4}\right) = \dfrac{1+\sin 6x}{2}$.

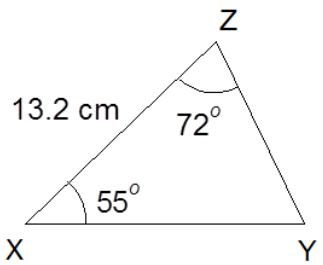
Trigonometric Equations

3. Solve $\sin 2x = \sin x$, where $0 \le x \le 2\pi$.
    (Attempt this question using two approaches: Factor and Double angle formula).

4. Solve $\sin^2 2x - \sin 2x = 2$, where $0 \le x \le 2\pi$.

Sine and Cosine Rule

5. The diagram (not drawn to scale) show triangle XYZ.
    Find the lengths of XY and YZ.



6. A triangle has side lengths 6 cm, 8 cm and 11 cm. Find the smallest angle in the triangle.

Overall

7. **NJC/Mid Year 2006/Q6**

(a) Prove the identity

$$2\cos\theta - \cos 3\theta - \cos 5\theta \equiv 16\cos^3\theta \sin^2\theta .$$ **[3]**

(b) Express $\mathrm{f}(\theta) = \cos^2\theta + \frac{1}{2}\sin 2\theta - 1$ in the form of $R\cos(2\theta - \alpha) - \frac{1}{2}$, where $R \in \mathbb{R}$ and $0 < \alpha < \frac{\pi}{2}$, giving the exact values of R and α. **[3]**

(c) Hence give the range of values of $\left[\mathrm{f}(\theta)\right]^2$. **[2]**

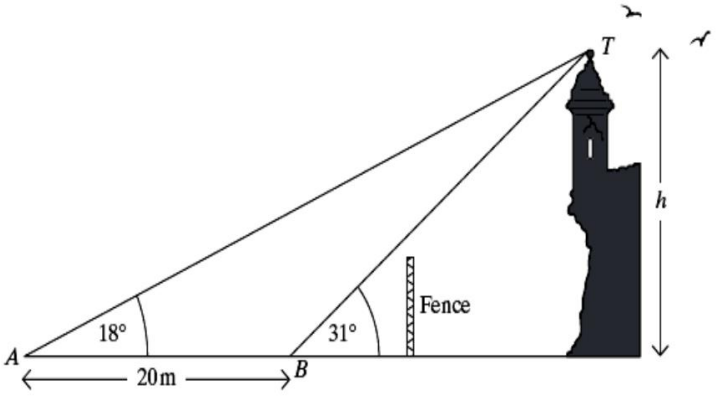
8. Consider a triangle ΔABC with $a > b > c$.

(i) What can you conclude about the relative sizes of sin *A*, sin *B* and sin *C*?

(ii) If no angle is obtuse, what can you conclude about the relative sizes of *A*, *B* and *C*?

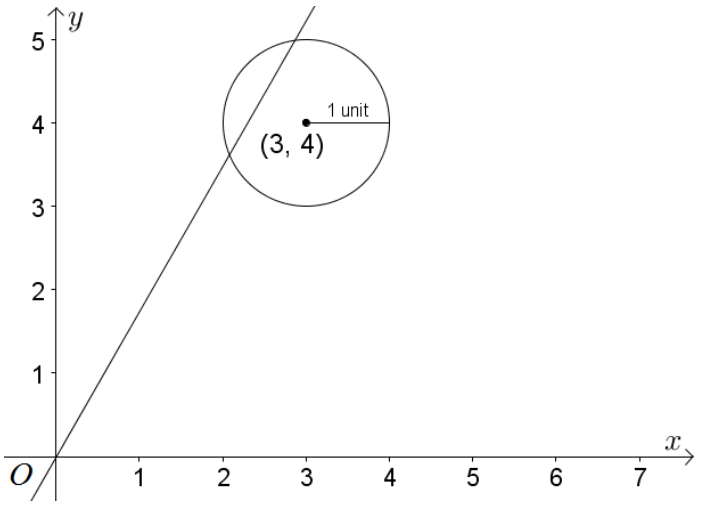
(iii) If $\angle A$ is obtuse, use the identity $\sin(180° - \theta) = \sin\theta$ to explain why $\sin A$ is larger than $\sin B$ and $\sin C$. Hence prove that $A > B > C$.

9. A ruined tower is fenced off for safety reasons. To find the height of the tower, Rachael stands at a point *A* and measures the angle of elevation as $18°$. She then walks 20 metres directly towards the base of the tower to point *B* where the angle of elevation is $31°$.



By finding $\angle ATB$, or otherwise, calculate the height, *h*, of the tower.

10. The following diagram shows a circle centred at (3, 4) with radius 1 unit, and a straight line that passes through the origin *O*.



Given that the straight line intersects the circle at least once, find the smallest and largest acute angle made by the line with the positive *x*-axis, giving your answers in radians correct to 3 decimal places.

Challenging Questions

1. Prove that $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ for $-1\le x\le 1$.

2. Solve $\sin x+\cos x=1$, where $0\le x\le 2\pi$.

Answers to Trigonometry Tutorial

Basic Mastery Questions

2. (a) 1 (b) 2 (c) 3

3. $\frac{\sqrt{2}}{2}$

4. (b)(i) $\frac{1}{x}$ (ii) $\sqrt{1+x^2}$ (iii) $\frac{\sqrt{1+x^2}}{x}$

5. 1, $\frac{4}{3}$

6. $2\sin\left(x+\frac{\pi}{3}\right)$

7. $x=\frac{\pi}{3}$ or $\frac{5\pi}{3}$

8. $x=\frac{5\pi}{6}$ or $\frac{11\pi}{6}$

9. $x=\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}$ or $\frac{15\pi}{8}$

10. (a) 23.6° (b) 16.0 cm (c) 6.01 cm
    (d) 89.6°

Practice Questions

1. $-\frac{33}{65}, \frac{63}{65}, -\frac{63}{65}$

3. $x=0,\ \pi,\ 2\pi,\ \frac{\pi}{3},$ or $\frac{5\pi}{3}$

4. $x=\frac{3\pi}{4}$ or $\frac{7\pi}{4}$

5. $\mathrm{XY}\approx 15.7\,\mathrm{cm},\ \mathrm{YZ}\approx 13.5\,\mathrm{cm}$

6. 32.2°

7. (b) $\frac{\sqrt{2}}{2}\cos\left(2\theta-\frac{\pi}{4}\right)-\frac{1}{2}$ (c) $0\le\left[\mathrm{f}(\theta)\right]^2\le\frac{2\sqrt{2}+3}{4}$

8. (i) $\sin A>\sin B>\sin C$

9. 14.2 metres

10. 0.726 rad, 1.129 rad

Challenging Questions

2. $x=0,\ \frac{\pi}{2}$ or 2π