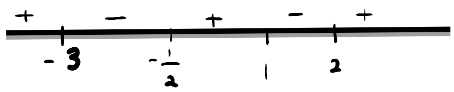
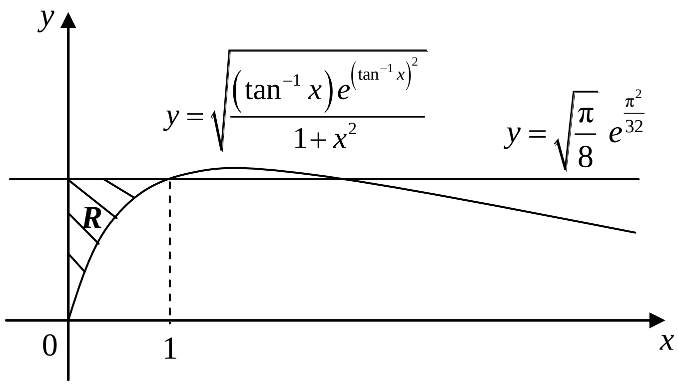
RVHS 2015 Y6 H2 MA Prelim Paper 1 (Solutions)

Question 1 [5 Marks]		
(a)	$\frac{7x-4}{x^2+2x-3} \ge 2$ $\frac{2x^2-3x-2}{x^2+2x-3} \le 0$ $\frac{(2x+1)(x-2)}{(x-1)(x+3)} \le 0$  $-3 < x \le -\frac{1}{2}$ or $1 < x \le 2$	
(b)	Sub $y = \lvert x \rvert$, $\frac{(2\lvert x\rvert+1)(\lvert x\rvert-2)}{\lvert x\rvert^2+2\lvert x\rvert-3} \le 0$ $\frac{(2\lvert x\rvert+1)(\lvert x\rvert-2)}{(\lvert x\rvert-1)(\lvert x\rvert+3)} \le 0$ $\frac{(2y+1)(y-2)}{(y-1)(y+3)} \le 0$ Therefore, from part (a) $-3 < y \le -\frac{1}{2}$ or $1 < y \le 2$ $-3 < \lvert x\rvert \le -\frac{1}{2}$ or $1 < \lvert x\rvert \le 2$ (No solution) $-2 \le x < -1$ or $1 < x \le 2$	

Question 2 [5 Marks]		
	Using cosine rule, $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos A\hat{B}C$ $AC = \left(1^2 + \left(\frac{1}{2}\right)^2 - 2(1)\left(\frac{1}{2}\right)\cos 2\theta\right)^{\frac{1}{2}}$ $= \left(1 + \frac{1}{4} - \left(1 - \frac{(2\theta)^2}{2!}\right)\right)^{\frac{1}{2}}$ $= \left(\frac{1}{4} + 2\theta^2\right)^{\frac{1}{2}}$ (shown) $= \sqrt{\frac{1}{4}}\left(1 + 8\theta^2\right)^{\frac{1}{2}}$ $\approx \frac{1}{2}\left(1 + \left(\frac{1}{2}\right)\left(8\theta^2\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(8\theta^2\right)^2\right)$ $= \frac{1}{2}\left(1 + 4\theta^2 - 8\theta^4\right)$ $= \frac{1}{2} + 2\theta^2 - 4\theta^4$	

Question 3 [8 Marks]		
(a)	$S_n = 2n^2 + 3n$ $S_{n-1} = 2(n-1)^2 + 3(n-1)$ $= (2n^2 - 4n + 2) + (3n - 3)$ $= 2n^2 - n - 1$ $u_n = S_n - S_{n-1}$ $= (2n^2 + 3n) - (2n^2 - n - 1)$ $= 4n + 1$ $u_{n-1} = 4(n-1) + 1$ $= 4n - 3$ $u_n - u_{n-1} = (4n+1) - (4n-3)$ $= 4$ Since $u_n - u_{n-1}$ = constant, the given sequence is an arithmetic progression. (shown) $S_n > 2015$ $2n^2 + 3n > 2015$ $2n^2 + 3n - 2015 > 0$ Using GC, $n < -32.5$ or $n > 31$ Since n is a positive integer, least value of n = 32.	
(b) (i)	Total height of an *n*-layered cake $= 10 + 10\left(\frac{5k}{100}\right) + 10\left(\frac{5k}{100}\right)^2 + \ldots + 10\left(\frac{5k}{100}\right)^{n-1}$ $= 10\left[1 + \left(\frac{k}{20}\right) + \left(\frac{k}{20}\right)^2 + \ldots + \left(\frac{k}{20}\right)^{n-1}\right]$ $= 10\left[\frac{1\left(1 - \left(\frac{k}{20}\right)^n\right)}{1 - \frac{k}{20}}\right]$	

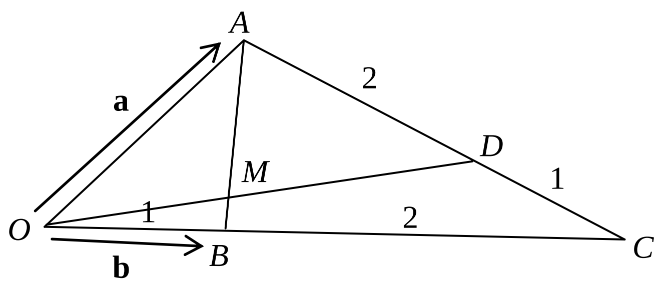
	$= \dfrac{200}{20-k}\left[1-\left(\dfrac{k}{20}\right)^n\right]$ (shown)	
(b) (ii)	When k = 19, Total height $= 200\left[1-\left(\dfrac{19}{20}\right)^n\right]$ $200\left[1-\left(\dfrac{19}{20}\right)^n\right] \le 120$ Using GC, $n \le 17.86375281$ Maximum number of layers is 17.	
(b) (iii)	It is assumed that the thickness of whipped cream used to join the different layers together is negligible.	

Question 4 [8 Marks]		
(a)	$\displaystyle\int \frac{\left(\tan^{-1} x\right)\mathrm{e}^{\left(\tan^{-1} x\right)^2}}{1+x^2}\,\mathrm{d}x$ $= \displaystyle\int u\mathrm{e}^{u^2}\,\mathrm{d}u$ (shown) $= \dfrac{1}{2}\mathrm{e}^{u^2} + C$ $= \dfrac{1}{2}\mathrm{e}^{\left(\tan^{-1} x\right)^2} + C$ $\boxed{\begin{aligned}\frac{dx}{du} &= \sec^2 u \\ &= 1+\tan^2 u \\ &= 1+x^2\end{aligned}}$	
(b) (i)		

(b) (ii)	Volume generated $= \pi \left(\sqrt{\frac{\pi}{8}}\ e^{\frac{\pi^2}{32}} \right)^2 (1) - \pi \int_0^1 \left(\sqrt{\frac{\left(\tan^{-1} x\right) e^{\left(\tan^{-1} x\right)^2}}{1+x^2}} \right)^2 \mathrm{d}x$ $= \frac{\pi^2}{8} e^{\frac{\pi^2}{16}} - \pi \left[\frac{1}{2} \mathrm{e}^{\left(\tan^{-1} x\right)^2} \right]_0^1$ $= \frac{\pi^2}{8} e^{\frac{\pi^2}{16}} - \frac{\pi}{2} \left[\mathrm{e}^{\frac{\pi^2}{16}} - 1 \right]$ $= \frac{\pi}{8} \left[\pi e^{\frac{\pi^2}{16}} - 4 e^{\frac{\pi^2}{16}} + 4 \right]$ units^3	

Question 5 [9 Marks]		
(a)	Let P_n denote the statement $u_n = \frac{n+1}{n!}$ for $n \geq 0$, $n \in$ □ . When $n = 0$, LHS $= u_0 = 1$ (given by question) RHS $= \frac{0+1}{0!} = 1 =$ LHS Therefore, P_0 is true. Assume that P_k is true for some $k \in$ □, $k \geq 0$. i.e. $u_k = \frac{k+1}{k!}$. We need to show that P_{k+1} is also true. i.e. $u_{k+1} = \frac{k+2}{(k+1)!}$. $u_{k+1} = u_k - \frac{k^2+k-1}{(k+1)!}$ $= \frac{k+1}{k!} - \frac{k^2+k-1}{(k+1)!}$ $= \frac{(k+1)^2 - k^2 - k + 1}{(k+1)!}$ $= \frac{k^2 + 2k + 1 - k^2 - k + 1}{(k+1)!}$ $= \frac{k+2}{(k+1)!}$ Therefore P_{k+1} is also true once P_k is true. Since P_0 is true, and P_k is true implies that P_{k+1} is also true, by Mathematical Induction, P_n is true for $n \in$ □, $n \geq 0$.	

(b)	$\displaystyle\sum_{r=1}^{N}\frac{(r+1)^2+r}{(r+2)!}=\sum_{j-1=1}^{N}\frac{j^2+j-1}{(j+1)!}$ (replacing r by $j-1$) $\displaystyle=\sum_{j=2}^{N+1}\frac{j^2+j-1}{(j+1)!}$ $\displaystyle=\sum_{j=2}^{N+1}\left(u_j-u_{j+1}\right)$ $=u_2-\cancel{u_3}$ $+\cancel{u_3}-\cancel{u_4}$ $+\cancel{u_4}-\cancel{u_5}$ $+\cancel{\ldots}$ $+\cancel{u_{N-1}}-\cancel{u_N}$ $+\cancel{u_N}-\cancel{u_{N+1}}$ $+\cancel{u_{N+1}}\quad-u_{N+2}$ $=u_2-u_{N+2}$ $\displaystyle=\frac{3}{2}-\frac{N+3}{(N+2)!}$	

Question 6 [9 Marks]		
(i)	$\lvert\mathbf{a}\times\mathbf{b}\rvert$ is the perpendicular distance of point A (with position vector $\mathbf{a}$) to line OB.	
(ii)		
(a)	$\overrightarrow{OC}=3\overrightarrow{OB}=3\,\mathbf{b}$ $\overrightarrow{OD}=\dfrac{2\overrightarrow{OC}+1\overrightarrow{OA}}{3}$ $=\dfrac{1}{3}(2(3\mathbf{b})+\mathbf{a})$ $=\dfrac{1}{3}\mathbf{a}+2\mathbf{b}$	

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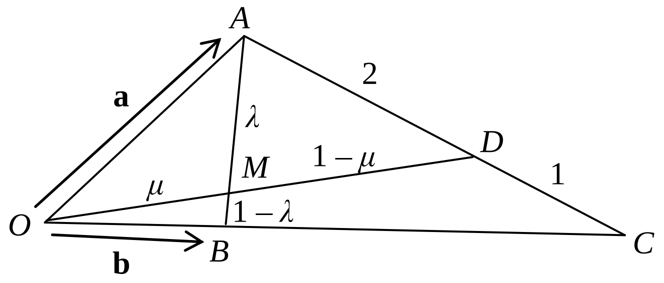

Let $\overrightarrow{AM} = \lambda\overrightarrow{AB}$,

$\overrightarrow{OM} = \overrightarrow{OA} + \lambda\overrightarrow{AB} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$ ---(1)

Let $\overrightarrow{OM} = \mu\overrightarrow{OD} = \dfrac{\mu}{3}\mathbf{a} + 2\mu\mathbf{b}$ ---(2)

Since **a** and **b** are non-zero and non-parallel,
(1) = (2)

$\Rightarrow 1-\lambda = \dfrac{\mu}{3},\ \lambda = 2\mu$

Solve simultaneously,

$\lambda = \dfrac{6}{7}, \quad \mu = \dfrac{3}{7}$

$\overrightarrow{OM} = \dfrac{1}{7}\mathbf{a} + \dfrac{6}{7}\mathbf{b}$
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<tr><td>(iii)</td><td>
Area of triangle OAC = $\dfrac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OC}\right|$

$12 = \dfrac{1}{2}\left|\mathbf{a}\times 3\mathbf{b}\right|$

$\Rightarrow \left|\mathbf{a}\times\mathbf{b}\right| = 8$

Shortest distance from A to OC

= $\left|\mathbf{a}\times\mathbf{b}\right|$

= $\left|\mathbf{a}\times\dfrac{\mathbf{b}}{\left|\mathbf{b}\right|}\right|$

= $\dfrac{1}{2}\times 8$ = 4 units
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Question 7 [10 Marks]

(i)

Method 1:

Let $z = x + \mathrm{i}y$ where $x, y \in \mathbb{R}$

$$z^2 = -8\mathrm{i}$$

$$x^2 - y^2 + 2xy\mathrm{i} = -8\mathrm{i}$$

Comparing real and imaginary parts,

$$x^2 - y^2 = 0 \quad \text{and} \quad 2xy = -8$$

$$x = \pm y \qquad\qquad xy = -4$$

When $x = y$, $y^2 = -4 \Rightarrow$ no solution

When $x = -y$, $-y^2 = -4 \Rightarrow y = \pm 2, x = \mp 2$

$\therefore z = 2 - 2\mathrm{i}$ or $-2 + 2\mathrm{i}$

Method 2:

$$z^2 = -8\mathrm{i}$$

$$z^2 = 8\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{2} + 2k\pi\right)} \text{ where } k \in \mathbb{Z}$$

$$z = 2\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4} + k\pi\right)} \text{ for } k = 0, 1$$

$$z = 2\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4}\right)} \text{ or } 2\sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{3\pi}{4}\right)}$$

$$z = 2 - 2\mathrm{i} \quad \text{or} \quad -2 + 2\mathrm{i}$$

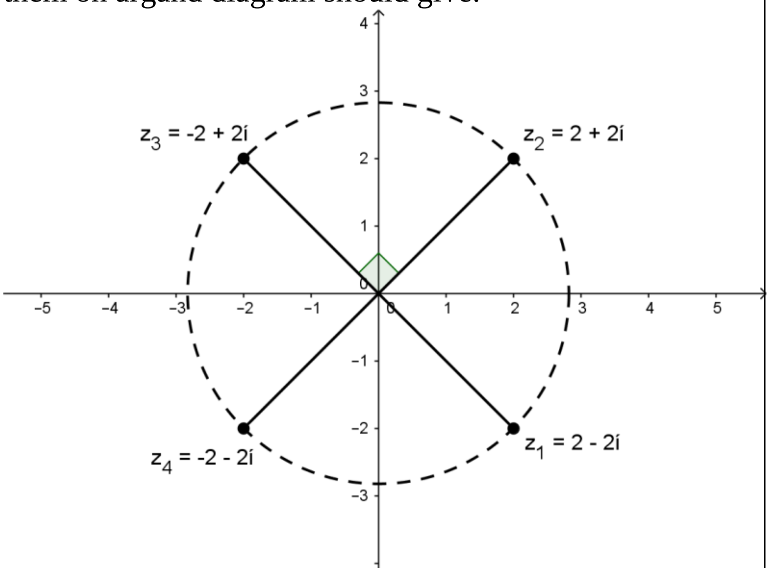
(ii)

$$w^4 = -64$$

$$w^2 = \pm 8\mathrm{i} \Rightarrow w^2 = 8\mathrm{i} \text{ or } w^2 = -8\mathrm{i}$$

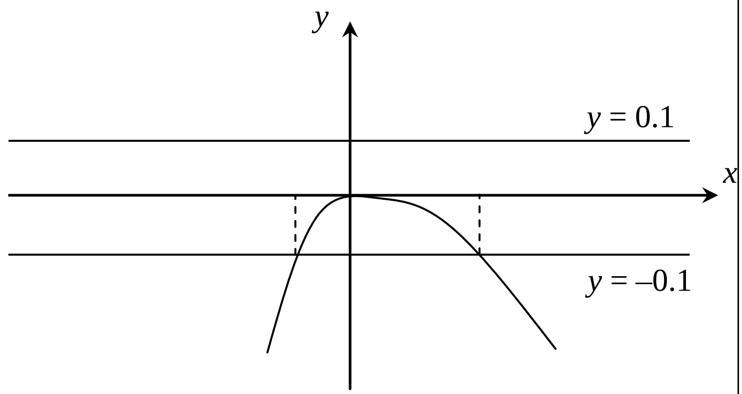
The roots of the second equation are the conjugates of those of the first.

Since two of the roots are those found in (i), representing them on argand diagram should give:



(iii)	$z^2+(2+2\mathrm{i})z+4\mathrm{i}=0$ $z=\dfrac{-(2+2\mathrm{i})\pm\sqrt{(2+2\mathrm{i})^2-4(1)(4\mathrm{i})}}{2}$ $=-1-\mathrm{i}\pm\dfrac{1}{2}\sqrt{-8\mathrm{i}}$ $=-1-\mathrm{i}\pm\dfrac{1}{2}(2-2\mathrm{i})$ $=-2\mathrm{i} \quad \text{or} \quad -2$	

Question 8 [10 Marks]		
	$y=\sin^{-1}[\ln(x+1)]$ --- (1) $\sin y=\ln(x+1)$ Differentiate implicitly with respect to x: $\cos y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x+1}$ (shown) --- (2) Differentiate implicitly with respect to x: $\cos y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\dfrac{\mathrm{d}y}{\mathrm{d}x}(-\sin y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{(x+1)^2}$ $\cos y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-\sin y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-\dfrac{1}{(x+1)^2}$ (shown) --- (3)	
(i)	Differentiate implicitly with respect to x: $\cos y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}(-\sin y)\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $-\left[(\sin y)(2)\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2\cos y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\right]=\dfrac{2}{(x+1)^3}$ $\cos y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}-3\sin y\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)-\cos y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3=\dfrac{2}{(x+1)^3}$ --- (4) When x = 0, (1): $y=\sin^{-1}0=0$ (2): $\cos(0)\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ (3): $\cos(0)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-\sin(0)(1)^2=-1\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-1$ (4):	

	$\cos(0)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}-3\sin(0)(-1)(1)-\cos(0)(1)^3=2\Rightarrow\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=3$ So, $y=0+1x+\dfrac{(-1)}{2!}x^2+\dfrac{3}{3!}x^3+\ldots$ $y=x-\dfrac{1}{2}x^2+\dfrac{1}{2}x^3+\ldots$	
(ii)	$\left\|y-\left(x-\dfrac{1}{2}x^2+\dfrac{1}{2}x^3\right)\right\|<0.1$ $-0.1<\sin^{-1}[\ln(x+1)]-\left(x-\dfrac{1}{2}x^2+\dfrac{1}{2}x^3\right)<0.1$ Sketch $y=\sin^{-1}[\ln(x+1)]-\left(x-\dfrac{1}{2}x^2+\dfrac{1}{2}x^3\right)$, $y=0.1$ and $y=-0.1$.  Solution set: $\{x:-0.521<x<0.783,\ x\in\mathbb{R}\}$	
(iii)	Consider $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}[\ln(x+1)]=\dfrac{1}{\sqrt{1-[\ln(x+1)]^2}}\dfrac{1}{x+1}$ $=\dfrac{1}{(x+1)\sqrt{1-(\ln(x+1))^2}}$ $\dfrac{1}{(x+1)\sqrt{1-(\ln(x+1))^2}}$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}[\ln(x+1)]$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x-\dfrac{1}{2}x^2+\dfrac{1}{2}x^3+\ldots\right)$ $\approx 1-x+\dfrac{3}{2}x^2$ (up to and including term in x^2)	

<table>
<tr><td colspan="3">Question 9 [11 Marks]</td></tr>
<tr><td>(i)</td><td>

$\overrightarrow{OA}=\begin{pmatrix}2\\0\\3\end{pmatrix},\quad \overrightarrow{OB}=\begin{pmatrix}-1\\-1\\4\end{pmatrix},\quad \overrightarrow{OC}=\begin{pmatrix}5\\1\\0\end{pmatrix}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}-3\\-1\\1\end{pmatrix}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}3\\1\\-3\end{pmatrix}$

Normal vector of Π_1 = $\overrightarrow{AB}\times\overrightarrow{AC}$

$=\begin{pmatrix}-3\\-1\\1\end{pmatrix}\times\begin{pmatrix}3\\1\\-3\end{pmatrix}$

$=\begin{pmatrix}2\\-6\\0\end{pmatrix}$

Since $\begin{pmatrix}2\\-6\\0\end{pmatrix}=2\begin{pmatrix}1\\-3\\0\end{pmatrix}$

Normal vector of Π_1 is parallel to $\mathbf{i}-3\mathbf{j}$,

$\mathbf{i}-3\mathbf{j}$ is perpendicular to the plane Π_1 (shown).

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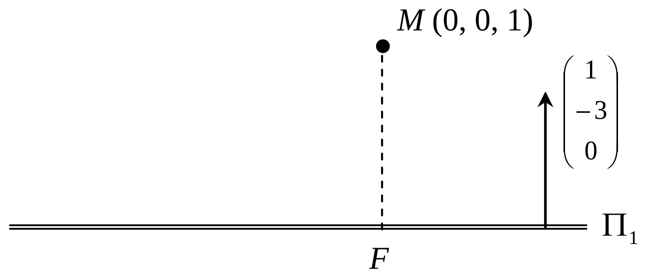
(ii) Let F be the foot of perpendicular of M to plane Π_1.



$$l_{MN}: \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}, \lambda \in R$$

$$\Rightarrow \overrightarrow{OF} = \begin{pmatrix} \lambda \\ -3\lambda \\ 1 \end{pmatrix} \text{ for some } \lambda \in \mathbb{R}$$

$$\Pi_1: \mathbf{r} \cdot \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} = 2$$

Since F lies on Π_1,

$$\begin{pmatrix} \lambda \\ -3\lambda \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} = 2 \Rightarrow \lambda + 9\lambda = 2$$

$$\Rightarrow \lambda = \frac{1}{5}$$

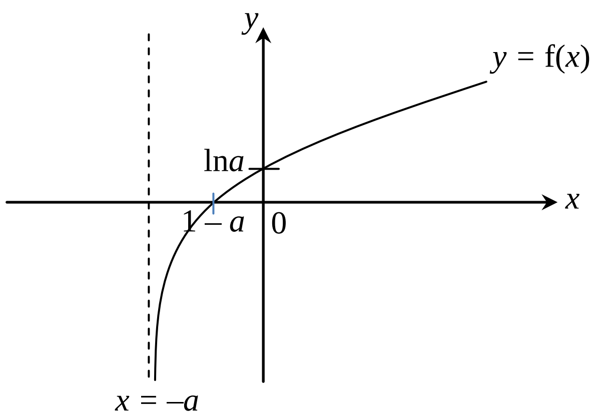
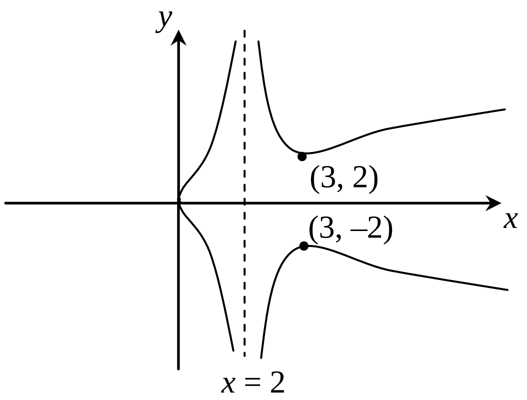
$$\overrightarrow{OF} = \begin{pmatrix} 1/5 \\ -3/5 \\ 1 \end{pmatrix}$$

$$\Rightarrow \overrightarrow{MF} = \begin{pmatrix} 1/5 \\ -3/5 \\ 0 \end{pmatrix}$$

So, distance $MF = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{3}{5}\right)^2 + (0)^2}$

$= \sqrt{\frac{2}{5}}$ units.

(iii)	Equation of Π_2: $ax - y + bz = 5 \quad \Rightarrow \quad \mathbf{r}.\begin{pmatrix} a \\ -1 \\ b \end{pmatrix} = 5$ Since M (0, 0, 1) lies on Π_2, $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} a \\ -1 \\ b \end{pmatrix} = 5$ So, $b = 5$ Since Π_2 is perpendicular to y-z plane, $\Rightarrow \Pi_2$ is parallel to $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} a \\ -1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$ So, $a = 0$ $\therefore \Pi_2 : \mathbf{r} \cdot \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix} = 5$ Let the acute angle between the planes be θ. $\left\lvert \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix} \right\rvert = \sqrt{10}\sqrt{26}\cos\theta$ $\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{10}\sqrt{26}} \right) = 79.3^\circ$ (1 d.p.)	

Question 10 [12 Marks]		
(a)	$y = \mathrm{f}(x) \xrightarrow{A} y = 2\mathrm{f}(x) \xrightarrow{B} y = -2\mathrm{f}(x)$, where *A*: stretch with scale factor 2 parallel to the *y*-axis, *B*: reflection about the *x*-axis. $-2\mathrm{f}(x) = \ln\left[\dfrac{1}{(x+a)^2}\right]$ $-2\mathrm{f}(x) = \ln(x+a)^{-2}$ $-2\mathrm{f}(x) = -2\ln(x+a)$ Therefore, $\mathrm{f}(x) = \ln(x+a)$  	
(b) (i)	$y^2 = \mathrm{g}(x)$  	

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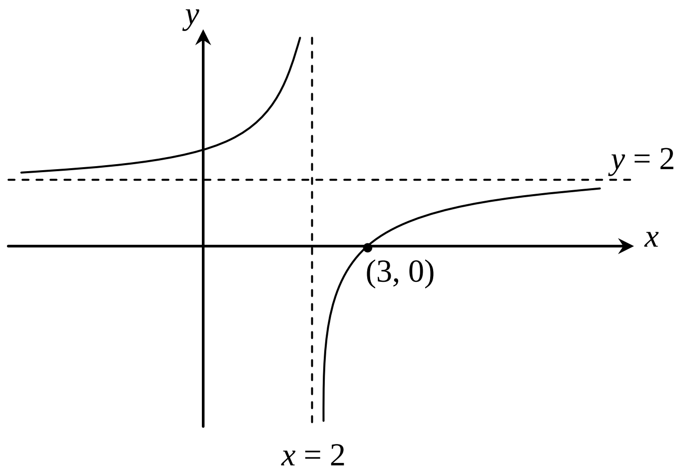
(ii)</td><td>$y = g'(x)$

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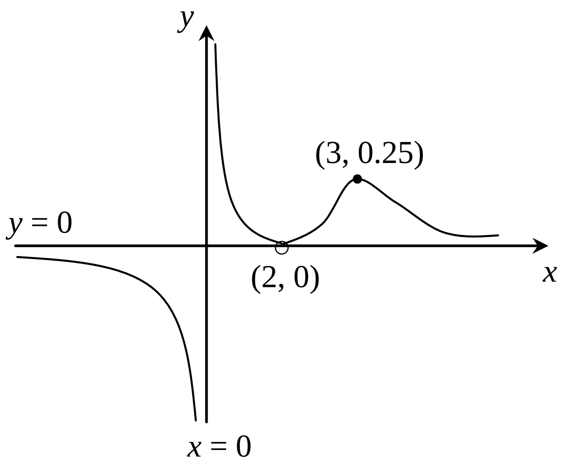
<tr><td>(b)
(iii)</td><td>$y = \dfrac{1}{g(x)}$

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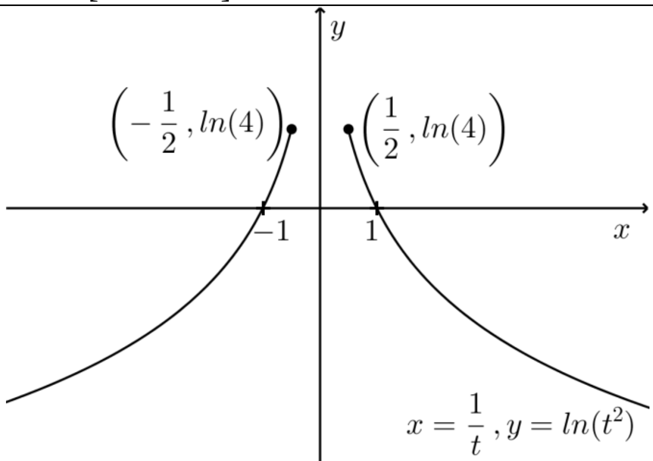
<tr><td></td><td>
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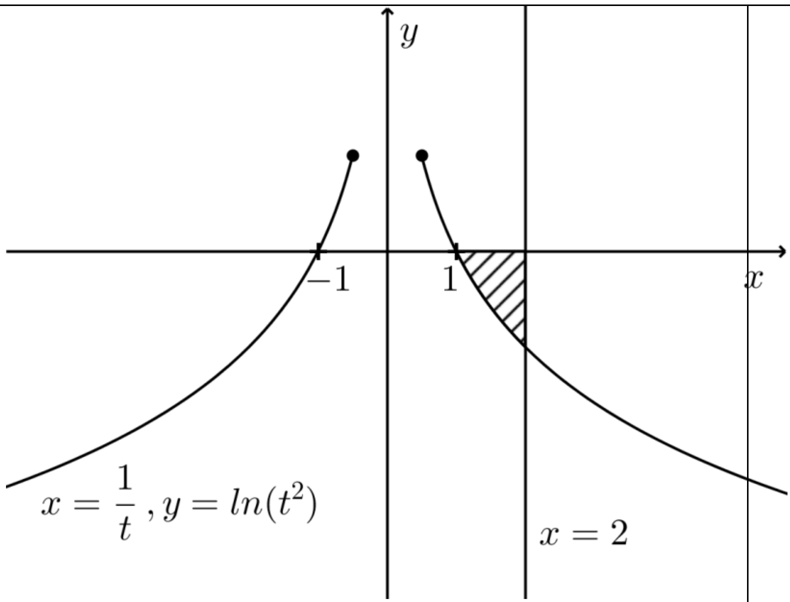
	$\frac{dy}{dt}=\frac{2}{t},\ \frac{dx}{dt}=-\frac{1}{t^2}$ $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=\frac{2}{t}\div\left(-\frac{1}{t^2}\right)=-2t$ At point P, $\frac{dy}{dx}=-2p$, gradient of normal $=\frac{1}{2p}$ ALTERNATIVELY $y=-2\ln x\ \Rightarrow\frac{dy}{dx}=\frac{-2}{x}\ \Rightarrow\left.\frac{dy}{dx}\right\|_{x=\frac{1}{p}}=-2p$ $\therefore$ gradient of normal $=\frac{1}{2p}$ Equation of normal is $y-\ln p^2=\frac{1}{2p}\left(x-\frac{1}{p}\right)$ $y=\frac{1}{2p}x-\frac{1}{2p^2}+\ln p^2$	
	At the point where the tangent does not intersect the normal at P, this tangent is parallel to the normal, $\frac{dy}{dx}=\frac{1}{2p}$ $-2t=\frac{1}{2p}$ $t=-\frac{1}{4p}$ When $t=-\frac{1}{4p}, x=-4p, y=-\ln\left(16p^2\right)$ $\therefore$ Coordinate of C are $\left(-4p,-\ln\left(16p^2\right)\right)$.	



Required area $= \left|\int_1^2 y\,\mathrm{d}x\right| = \left|\int_1^{\frac{1}{2}} y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t\right|$

$= \left|\int_1^{\frac{1}{2}} \left(\ln t^2\right)\left(-\frac{1}{t^2}\right)\mathrm{d}t\right|$

$= \left|\left[\frac{1}{t}\ln t^2\right]_1^{\frac{1}{2}} - \int_1^{\frac{1}{2}} \frac{2}{t^2}\,\mathrm{d}t\right|$

$= \left|-2\ln 4 - 0 + 2\left[\frac{1}{t}\right]_1^{\frac{1}{2}}\right|$

$= \left|2 - 2\ln 4\right| = 2\ln 4 - 2$ units^2

<u>ALTERNATIVELY</u>

$y = \ln\left(\frac{1}{x^2}\right) = -2\ln x$

Required area $= \left|\int_1^2 y\ \mathrm{d}x\right|$

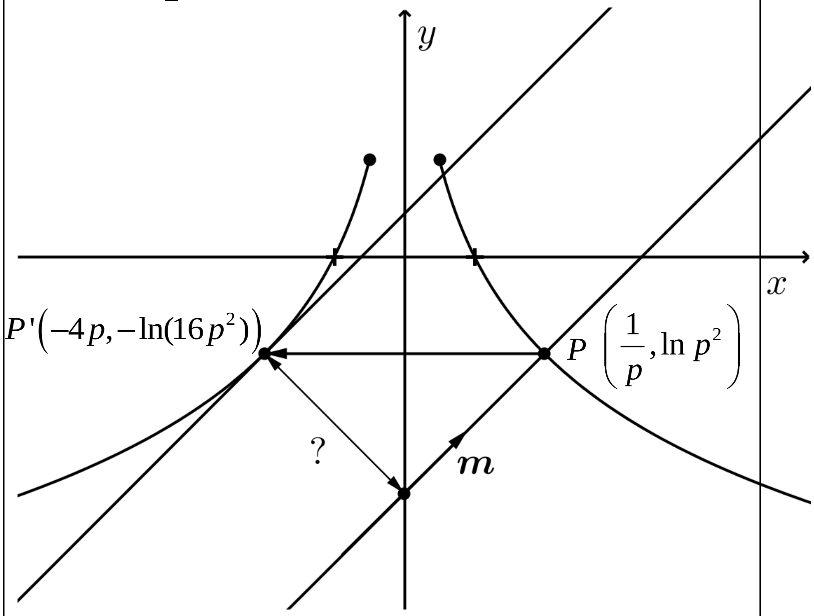
$= \left|-2\int_1^2 \ln x\ \mathrm{d}x\right|$

$= 2\left|\left[x\ln x\right]_1^2 - \int_1^2 1\ \mathrm{d}x\right|$

$= 2\left|2\ln 2 - 0 - (2-1)\right|$

$= 4\ln 2 - 2$ units^2

When $p = \dfrac{1}{2}$,



Direction vector parallel to normal, **m**, is $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

Coordinates of point *P* and corresponding point, *P*', where tangent does not intersect the normal at P are $(2, -\ln 4, 0)$ and $(-2, -\ln 4, 0)$ respectively.

Distance between the normal and tangent

$$= \left| \overrightarrow{PP'} \times \mathbf{m} \right|$$

$$= \left| \left[\begin{pmatrix} -2 \\ -\ln 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -\ln 4 \\ 0 \end{pmatrix} \right] \times \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right|$$

$$= \frac{1}{\sqrt{2}} \left| \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right| = \frac{1}{\sqrt{2}} \left| \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} \right| = 2\sqrt{2} \text{ units}$$