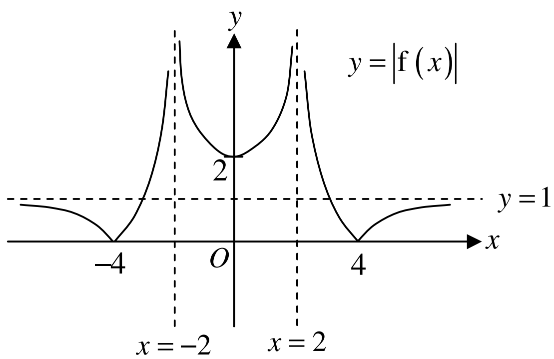
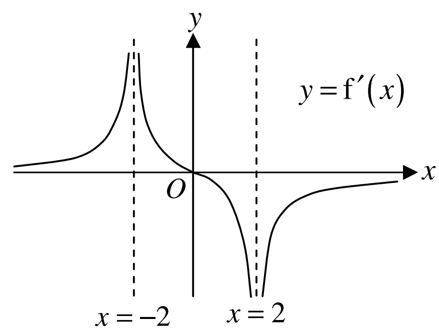
Section A: Pure Mathematics [40 marks]

1 The graph of $y=|\mathrm{f}(x)|$ has a turning point at $(0,2)$ and passes through the points $(-4,0)$ and $(4,0)$. The graph of $y=\mathrm{f}'(x)$ has a point of inflexion at the origin.





On separate diagrams, sketch the graphs of

(i) $y=\mathrm{f}'(-2x)$, [2]

(ii) $y=\mathrm{f}(x)$, [3]

stating the equations of any asymptotes, the coordinates of any stationary points and points of intersection with the axes.

2 **(i)** By using the substitution $x=\frac{1}{u}$, show that $\int_{2\sqrt{2}}^{4} \frac{1}{x\sqrt{x^2-4}}\,\mathrm{d}x=\frac{\pi}{24}$. [4]

(ii) O is the origin and A is a point on the curve $y=\frac{1}{x\sqrt{x^2-4}}$ where $x=2\sqrt{2}$. The region R is enclosed by the curve $y=\frac{1}{x\sqrt{x^2-4}}$, the line OA, the line $x=4$ and the x-axis. Find the exact area of R. [4]

3 A sequence of positive numbers $x_1,\ x_2,\ x_3,\ \ldots$ satisfies the recurrence relation

$$x_{n+1}=\sqrt{10-3x_n}\,,\ \text{for } n=1,\ 2,\ 3,\ \ldots.$$

(i) Given that the sequence converges to l, find the value of l. [2]

(ii) Prove that $\left(x_{n+1}\right)^2-l^2=3\left(l-x_n\right)$. [2]

(iii) Use the result in part **(ii)** to show that if $x_n<l$, then $x_{n+1}>l$. [2]

4 A sector of angle θ radians and radius 4π cm is used to form the curved surface of a right circular cone such that there is no overlapping. Prove that the volume of the cone is

$$\frac{8\pi}{3}\sqrt{4\pi^2\theta^4-\theta^6}\ \text{cm}^3. \qquad [3]$$

Show that the maximum volume of the cone is $\dfrac{p\sqrt{3}}{q}\pi^4$ cm^3 as θ varies, where p and q are integers to be determined. [5]

[The formula for the arc length of a sector of a circle is $s = r\theta$.]

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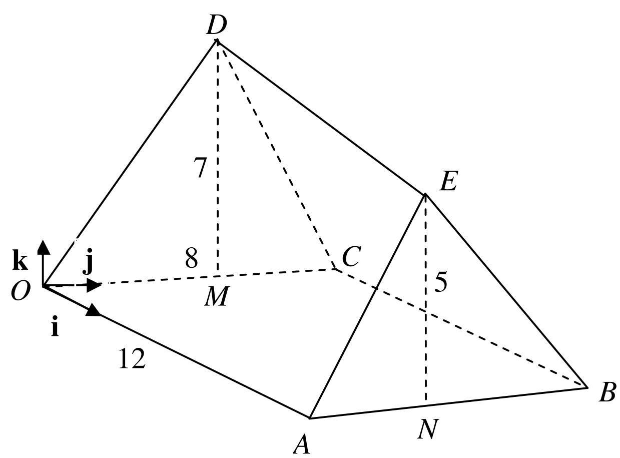

The diagram shows a solid with a horizontal rectangular base $OABC$ in which the lengths of OA and OC are 12 m and 8 m respectively. M and N are the mid-points of OC and AB respectively, and D and E are 7 m and 5 m vertically above M and N respectively. The point O is taken as the origin for position vectors. Perpendicular vectors **i**, **j, k** are such that **i** and **j** are parallel to OA and OC respectively, and **k** is perpendicular to the plane $OABC$.

(i) Find a vector equation of the line DB. Hence find the foot of perpendicular from E to the line DB. [5]

(ii) Find the acute angle between line DB and the plane OBE. [4]

(iii) Find the length of projection of DE onto DB. [3]

(iv) State, giving a reason, whether lines DB and AC are coplanar. [1]

Section B: Statistics [60 marks]

6 The town council of a particular housing estate wishes to find out whether the residents of the housing estate want to have a wet market built. A survey is to be carried out on 10% of the households in the housing estate.

(i) Explain how a systematic sample might be carried out. [2]

(ii) Describe one other method of sampling that could be used, and state one advantage that this method has over systematic sampling. [2]

7 A die is biased and the probability, p, of throwing a six is known to be less than $\frac{1}{6}$. An experiment consists of recording the number of sixes in 25 throws of the die. In a large number of experiments, the standard deviation of the number of sixes is 1.5. Show that the value of p is $\frac{1}{10}$. Hence find the probability that at least 6 but fewer than 10 sixes are recorded during a particular experiment. [4]

The biased die is now thrown 40 times. Find the most likely number of sixes obtained. [1]

8 Call-outs at fire stations are classified as either genuine or false, and occur at random times. All call-outs are independent of one another.

(a) At a fire station in town A, on average there are two genuine call-outs in a week, and one false call-out in a two-week period.

(i) Find the probability that there are fewer than 6 genuine call-outs in a randomly chosen two-week period. [2]

(ii) Using a suitable approximation, estimate the probability that the total number of call-outs in a randomly six-week period exceeds 19. [4]

(b) At a fire station in town B, on average there are m genuine call-outs in a week. Given that the probability of at most one genuine call-out in a randomly chosen week is 0.08, write down an equation for the value of m and find this value numerically. [2]

9 The mileage, X km per litre of petrol, of a particular model of car is a random variable with mean μ km per litre. A petroleum company had modifications done to their car petrol with the intention of improving mileage in cars. The company conducted a pilot test of the modified petrol with 13 cars of that particular model. The results are summarised by

$$\sum(x-12)=6.09, \qquad \sum(x-12)^2=20.853.$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) It is given that $\mu=\mu_0$ for the unmodified petrol. A test is carried out at the 5% significance level to determine if the modified petrol improves mileage in cars. Find the set of values of μ_0 for the company to have insignificant evidence that the modified petrol improves mileage in cars. [4]

(iii) Based on observations over a long period, it is found that that $\mu=12$. Find the least significance level for the company to have significant evidence that the modified petrol improves mileage in cars. [2]

State an assumption which you need to make for the above tests to be valid. [1]

10 A class of twenty-five pupils consists of 15 girls and 10 boys. At the beginning of the year four pupils are to be chosen at random to form an interim class committee comprising of "Chairperson", "Vice-Chairperson", "Treasurer" and "Secretary". Find

(i) the probability that exactly two girls are chosen, [2]

(ii) the probability that the Chairperson and Vice-Chairperson are of opposite sex, [2]

(iii) the probability that the Chairperson and Vice-Chairperson are both boys given that the Treasurer and Secretary are of opposite sex. [3]

State with a reason whether or not the events 'Chairperson and Vice-Chairperson are both boys' and 'Treasurer and Secretary are of opposite sex' are independent. [2]

11 An athletics coach believes that athletes with longer legs can run faster. He selected 10 of his athletes and recorded their leg lengths, x metres and their timings, t seconds, in a 100 m race. The results are given in the table.

x	0.70	0.76	0.80	0.84	0.85	0.89	0.92	0.95	0.98	1.00
t	13.90	12.73	12.12	11.89	11.80	11.42	11.29	10.94	11.00	10.80

It is given that the value of the product moment correlation coefficient for this data is -0.963, correct to 3 decimal places.

(i) State, with a reason, whether the value of the product moment correlation coefficient would be different if the leg lengths had been measured in centimetres instead. [1]

(ii) One of the athletes, Aaron, had missed the race. Assuming a linear model, the coach decides to use a regression line to estimate Aaron's 100 m race timing by measuring his leg length. Explain which of the least squares regression lines, x on t or t on x, should be used. [1]

(iii) Draw a scatter diagram to illustrate the data. [2]

(iv) Aaron disagreed with the coach and claimed that that x and t do not have a linear correlation. Comment on Aaron's statement with reference to the scatter diagram. [1]

(v) To be fair to Aaron, the coach considered another possible model for the relationship between x and t:

$$t = a + \frac{b}{x^2},$$

where a and b are constants.

(a) Find the value of the product moment correlation coefficient between t and $\frac{1}{x^2}$, and hence explain why this new model is better than the linear model. [2]

(b) The coach wants to train an athlete to run the 100 m race in 10 seconds. Calculate the equation of the regression line based on the new model, and use it to estimate the minimum leg length required for the potential athlete. Comment on the reliability of the estimate. [4]

12 A factory manufactures paperweights consisting of glass mounted on a wooden base. The volume of glass, in cm^3, in a randomly chosen paperweight has a normal distribution with mean 56.5 and standard deviation 2.9. The volume of wood, in cm^3, has an independent normal distribution with mean 38.4 and standard deviation σ.

The probability that the total volume of a randomly chosen paperweight exceeds 100 cm^3 is 0.05. Show that σ is 1.10, correct to 3 significant figures. [4]

(i) Find the probability that the mean volume of glass per paperweight in a random sample of 20 paperweights is less than 57.1 cm^3. [2]

(ii) Find the probability that two paperweights selected at random have volumes of wood which differ by at least 0.07 cm^3. [3]

The glass weighs 3.1 grams per cm^3 and the wood weighs 0.8 grams per cm^3.

(iii) Find the probability that the total mass of a randomly chosen paperweight is between 200 and 220 grams. [4]

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