Candidate Name : ______________________________ CT Group : _________

Index no : _________

PIONEER JUNIOR COLLEGE
JC 2 Preliminary Examination

MATHEMATICS
Higher 2 Paper 1

(9740 / 1)

Wednesday **14 Sept 2011**

Additional material: Answer paper, List of Formulae MF15

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your full name, index number and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
Attach this question paper with your answers, and arrange your answers in numerical order.

For Examiner's Use					
Qn	**Marks**	**Qn**	**Marks**	**Qn**	**Marks**
1		5		9	
2		6		10	
3		7		11	
4		8		12	
Sub-total		Sub-total		Total	

This question paper consists of 7 printed pages and 1 blank page.

1 **(a)** Find $\int x\tan^{-1}\left(2x^2\right)\mathrm{d}x$. [3]

(b) The region R is bounded by the curve $y = 4x - x^2$, the line $2y = 9 - x$, and the y-axis. Find the volume of the solid when R is rotated completely about the x-axis, giving your answer correct to 2 decimal places. [3]

2 **(i)** Find the first three terms in the expansion of $\dfrac{1}{\sqrt{4+x^2}}$ in ascending powers of x. [3]

(ii) Hence find the first four terms in the expansion of $\dfrac{x+1}{\sqrt{4+x^2}}$. [2]

(iii) State the set of values of x for which this expansion is valid. [1]

3 Each time a ball falls vertically onto a horizontal floor, it rebounds to three - quarters of the height from which it fell. It is initially dropped from a point 4 m above the floor.

(i) Show that the total distance the ball travels until it is about to touch the floor for the $(n+1)$th time is given by $28 - 24\left(\dfrac{3}{4}\right)^n$. [3]

(ii) Find the least number of times the ball must bounce for it to travel more than 24 m. [2]

(iii) Explain why the ball will not travel more than 28 m. [1]

4 **(i)** By writing $\dfrac{1}{(r-3)(r-2)}$ in partial fractions, find $\displaystyle\sum_{r=4}^{N}\frac{1}{(r-3)(r-2)}$.

[4]

(ii) Hence find $\displaystyle\sum_{r=0}^{N-6}\frac{1}{(r+2)(r+1)}$. [2]

(iii) Give a reason why the series $\displaystyle\sum_{r=4}^{\infty}\frac{1}{(r-3)(r-2)}$ converges, and write down its value. [2]

5 Given that $y = \mathrm{e}^x \cos^2 x$, prove that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = -2\mathrm{e}^x \cos 2x$. [3]

By further differentiation of this result, obtain the series expansion of y in ascending powers of x up to and including the term in x^3. [3]

Given that x is sufficiently small for x^4 and higher powers of x to be neglected, deduce the series expansion of $\mathrm{e}^x \cos x \sin 2x$. [2]

6 The functions f and g are defined as follows:

$$\mathrm{f}: x \mapsto \frac{1-2x}{x-2}, \qquad x < 2,$$
$$\mathrm{g}: x \mapsto \ln(x+3), \quad x > -3.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$, showing clearly the asymptote(s) of the graph. [2]

(ii) Find the expression for $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(iii) Only one of the composite functions fg and gf exists. Give a definition (including the domain) of the composite that exists, and explain why the other composite does not exist. [3]

7 A curve is defined by the parametric equations $x = \frac{a}{t^3}$, $y = \frac{a}{t}$ where a is a constant.

(i) Find the equations of the tangent and the normal at point P where $t = \frac{1}{2}$. [4]

(ii) Find the coordinates of the point where the tangent cuts the curve again. [2]

(iii) The tangent at P meets the x-axis at Q and the normal at P meets the x-axis at R. Show that the area of triangle PQR is $\frac{145}{6}a^2$. [2]

8 In a chemical plant, the amount of substance X in a chemical reaction is being observed. The amount of substance X, in kg at any time t minutes after the start of the chemical reaction is denoted by x and it satisfies the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = k\left(1-2x\right)$, where k is a positive constant. It is known that at the start of the chemical reaction, the amount of substance X is 1 kg and is decreasing at a rate of 0.05 kg per minute.

(i) Obtain an expression for x in terms of t. [4]

(ii) Using a non-graphical method, show that the amount of substance X is always decreasing. [2]

(iii) State, with a reason whether substance X will be used up in the long run. [1]

(iv) Sketch a graph to show how the amount of substance X varies with time. [2]

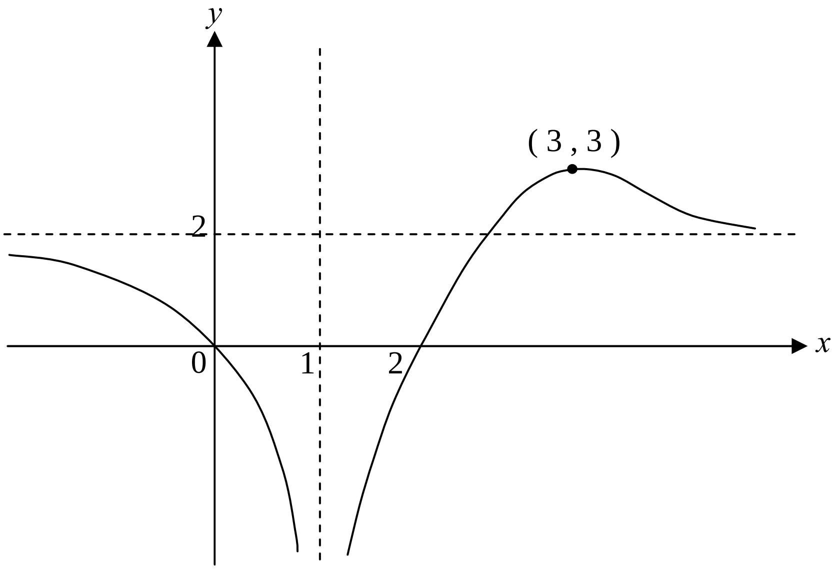
9 The polynomial $\mathrm{P}\left(z\right)$ has real coefficients. The equation $\mathrm{P}\left(z\right)=0$ has a root $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leq\pi$.

(i) Write down a second root in terms of r and θ, and hence show that a quadratic factor of $\mathrm{P}\left(z\right)$ is $z^2-2rz\cos\theta+r^2$. [3]

(ii) z_1 and z_2 are roots of the equation $\mathrm{P}\left(z\right)=0$, where $z_1=2\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$ and $z_2=\mathrm{i}z_1$. Write down the exact modulus and argument of z_2. State the geometrical relationship between z_1 and z_2 and illustrate this relationship clearly on an Argand diagram. [3]

(iii) Given further that $\mathrm{P}\left(z\right)$ is of degree four, express $\mathrm{P}\left(z\right)$ as a product of two quadratic factors with real coefficients, giving each factor in exact non - trigonometrical form. [3]

10 **(a)** The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph crosses the x-axis at $x = 0$, $x = 2$ and has a turning point at $(3, 3)$. The asymptotes of the graph are $x = 1$ and $y = 2$.



Sketch, on separate clearly labelled diagrams, the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y = \sqrt{\mathrm{f}(x+1)}$. [3]

(b) A graph with the equation $y = \mathrm{f}(x)$ undergoes, in succession, the following transformations:

A: A translation of 1 unit in the direction of the x-axis.

B: A stretch parallel to the x-axis by a scale factor $\dfrac{1}{2}$.

C: A reflection in the y-axis.

The equation of the resulting curve is $y = \dfrac{4}{4x^2 + 4x + 1}$.

Determine the equation of the graph $y = \mathrm{f}(x)$, giving your answer in the simplest form. [4]

11 The curve C has equation $y=\dfrac{x^2-4x+k^2}{x-k}$, $x\neq k$, and k is a constant such that $k\neq 0$ and $k\neq 2$.

(i) Find the equations of the asymptotes of C. [2]

(ii) Show that if C has 2 stationary points, then $k<0$ or $k>2$. [3]

(iii) Given that $y=x$ is an asymptote of C, find the value of k. With this value of k, sketch C, showing clearly the asymptotes and the stationary points. [3]

(iv) By adding a suitable graph which passes through the point $(4,4)$ on the sketch of C, find the range of values of p for which the equation $\left(x^2-4x+k^2\right)-(x-k)(px+4-4p)=0$ has exactly 2 real roots for the value of k found in **(iii)**. [2]

12 The position vectors of the points A, B, C are $\begin{pmatrix}0\\2\\-3\end{pmatrix}$, $\begin{pmatrix}3\\3\\-3\end{pmatrix}$, $\begin{pmatrix}-9\\2\\6\end{pmatrix}$ respectively.

(i) Given that A, B, C lies on plane Π_1, show that the equation of Π_1 is

$$\mathbf{r}\cdot\begin{pmatrix}1\\-3\\1\end{pmatrix}=-9.$$ [2]

The equations of the plane Π_2 and the line l are $\mathbf{r}\cdot\begin{pmatrix}4\\-2\\1\end{pmatrix}=-6$ and $\mathbf{r}=\begin{pmatrix}1\\1\\4\end{pmatrix}+\mu\begin{pmatrix}-1\\-2\\3\end{pmatrix}, \mu\in\mathbb{R}$ respectively.

(ii) Find the position vector of the foot of the perpendicular from C to the line l. [3]

(iii) Find a vector equation of the line of intersection between Π_1 and Π_2. [2]

(iv) The plane Π_3 has the Cartesian equation $ax + by + cz = 3$, where a, b and c are constants. Π_1, Π_2 and Π_3 intersect in a line, and the point with position vector $\begin{pmatrix} 5 \\ -12 \\ 2 \end{pmatrix}$ lies on Π_3. Find a, b and c.

Hence find the acute angle between Π_3 and l. [5]