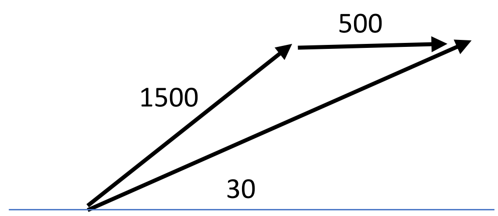
Q1 (a) 4u(2000) = (4u+12u)V_{cm}

V_{cm}= (4/16)2000 = 500 m/s [1]

(b)



[1]

$V_{\alpha,cm} = 2000 - 500 = 1500\ ms^{-1}$ [1]

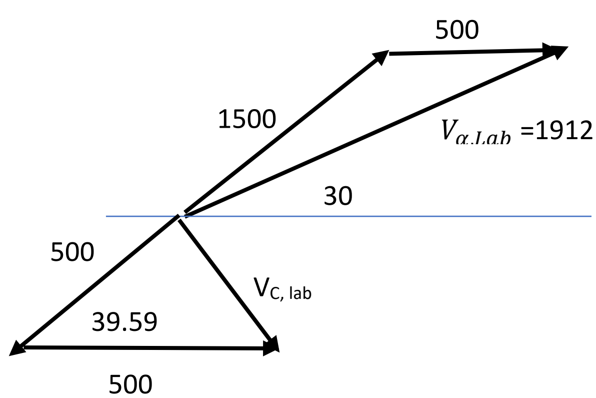
$\frac{\sin(\alpha)}{500} = \frac{\sin(30)}{1500}, \alpha = 9.59^o$, total = 39.59 from horizontal

$V_{\alpha,Lab} = \sqrt{1500^2 + 500^2 - 2\ x\ 1500x500xcos(180 - 9.59 - 30)} = 1912\ ms^{-1}$ [2]

(c) $V_{C,cm} = 0 - 500 = -500\ ms^{-1}$ [1]

Angle between C and V_{cm}= 180-39.6-(180-39.6)/2 =70.2° (opposite side of alpha)

$V_{C,lab} = \sqrt{500^2 + 500^2 - 2x500x500\cos(39.59)} = 338.7\ ms^{-1}$ [2]



Q2 (a) Let mass = M = $\rho\pi(4R^2 - R^2) = 3\rho\pi R^2$

Circular sheet can be taken as a complete disc with density of ρ and radius 2R (mass of 4M/3), superimposed with a disc of R with density of – ρ or mass of -M/3 [1]

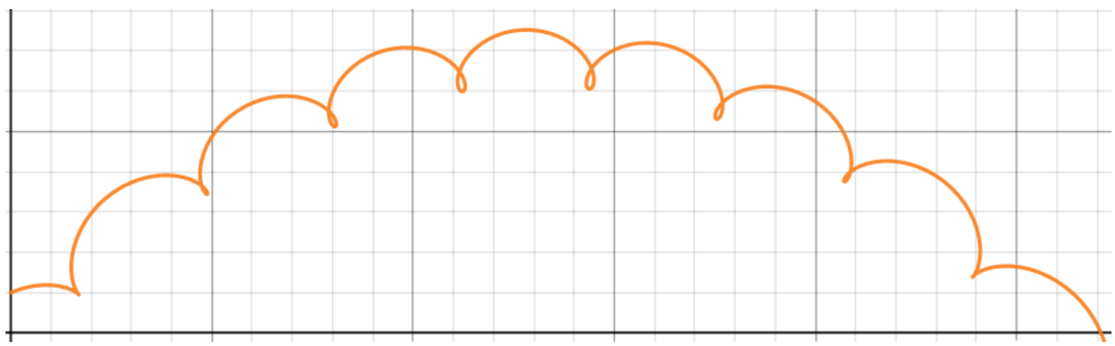
X-position of CM = at vertical mid-line of circular disc

Y-position of CM: Y = $\frac{\left[\left(\frac{4M}{3}\right)2R-\left(\frac{M}{3}\right)3R\right]}{M} = \frac{5R}{3}$ from bottom of circular sheet. [1]

(b) (i) Path shows normal projectile/parabolic path of the CM with loops in the forward direction. [1]

The separation of the loops in the x-directions are equal. [1]

The separation of the loops in the y-direction are closer nearer the top. [1]



(ii) The horizontal position x= 22.6cos(45)t+(7R/3)sin(wt).

The vertical position y= 22.6sin(45)t +(7R/3)cos(wt) + 5R/3- ½ .9.81 t^2 [1]

Time of flight can be estimated by ignoring the 2^{nd} and 3^{rd} term y which are much smaller in magnitude

Estimated time of flight = 2x 22.6sin(45) /9.81 = 3.3 s or 1.65 s to the highest point [1]

The horizontal velocity is $22.6\cos(45) + 7R(5\pi)\cos(5\pi t)/3$ = 16+21.98$\cos(5\pi t)$ [1]

Move backwards means horizontal velocity < 0

Solving for 16+21.98$\cos(5\pi t) < 0$, for t values less than 1.65 s, T = 0.4

gives t = 0.152s, 0.152+0.4=0.552, 0.552+0.4=0.952, 0.952+0.4=1.35, 1.35+0.4=1.75 (exceed)

So the outer most edge moves back 4 times before reaching the top [1]

(iii) At t = 0.152 s, vertical velocity = 22.6sin(45) -(7Rw/3)sin(wt) -9.81 t

= 16 -15.08 -2.43 m/s = -1.51 m/s [1]

Q3 (a)(i) Speed at start = 2x3.14x1.496x10^{11}/3.256x10^7=29768 m/s

Speed of venus orbit = 2x3.14 x 0.7233x1.496x10^{11}/1.941x10^7= 35009 m/s [1]

Semi-major axis of transfer orbit = (1+0.7233)/2 AU = 1.289 x 10^{11} m [1]

$$\frac{GMm}{r^2} = mr\left(\frac{4\pi^2}{T^2}\right), GM = \frac{4\pi^2 r^3}{T^2} = 1.326\ x10^{20}$$

At start of transfer $\frac{1}{2}mv_E^2 - \frac{GMm}{1.496x\ 10^{11}} = -\frac{GMm}{2x1.289x10^{11}}$

$v_E = \sqrt{2GM\left(\frac{1}{1.497x10^{11}} - \frac{1}{1.289x10^{11}}\right)}$ = 27276 m/s

Reduction of speed at LEO = 29768 – 27276 = 2490 m/s [1]

At destination: $\frac{1}{2}mv_V^2 - \frac{GMm}{0.7233x1.496x\ 10^{11}} = -\frac{GMm}{2x1.289x10^{11}}$

V_V = 37711 m/s, Reduction in speed at venus orbit = 37711 -35009

= 2700 m/s [1]

(b) Transfer time = $\frac{1}{2}\sqrt{\frac{4\pi^2 a^3}{GM}} = \frac{1}{2}\frac{\sqrt{4x3.14^2x(1.289x10^{11})^3}}{\sqrt{(GM)}}$=1.26x$10^7$s = 146 days [1]

(c) Venus turn through angle of 360x(1.26x10^7/1.941 x 10^7) = 233.7 [1]

Spacecraft from Earth turns through 180 degree

Angle between Earth-Venus at start = 233.7 -180 = 53.7° [1]

4 (a) $V_c = E\left(1 - \exp\left(-\frac{t}{RC}\right)\right)$

$\frac{E}{2} = \quad E\left(1 - \exp\left(-\frac{t}{RC}\right)\right)$

Simplifies to t = RC ln(2) [2]

(b) (i) Effective inductance = 2L + 2M [1]

$f = \frac{1}{2\pi\sqrt{(2L+2M)C}}$ [1]

(ii) $(2L + 2M)\left(\frac{dI}{dt}\right) + IR + \frac{q}{C} = 0$ [1]

$$(2L + 2M)\left(\frac{d^2q}{dt^2}\right) + R\left(\frac{dq}{dt}\right) + \frac{q}{C} = 0$$

Substitute $q = Q_o \exp(-\gamma t)\sin(\omega t + \phi)$ into above equation: [1]

Group terms in cos: $\gamma = \frac{R}{2(2L+2M)}$ [1]

Group terms in sin: $\gamma^2 - \omega^2 - \frac{R}{2L+2M}\gamma + \frac{1}{(2L+2M)C} = 0$

$$\omega = \sqrt{\frac{1}{(2L+2M)C} - \frac{R^2}{4(2L+2M)^2}}$$ [1]

5(a) E=½ $CV^2 = \frac{1}{2}\frac{\epsilon_o A}{d}V^2$ [1]

$= 6.91 \times 10^{-7}$ J [1]

(b) $E = \frac{1}{2}\frac{\epsilon_o A}{d}V^2 \propto \frac{1}{d}$, new E = $6.91 \times 10^{-7}/2 = 3.45 \times 10^{-7}$J [1]

Less energy stored. Work done by hand and energy lost by capacitor because charge flows back to charge battery. [1]

(c) Energy = ½ Q^2/C = ½ $(C_oV)/(C_o/2) = 1.38 \times 10^{-6}$J [1]

The increase in energy is accounted for by the work done in pulling the capacitor plates. [1]

(d) Energy stored decrease because of higher capacitance [1]

Energy stored reduces because work is done by the capacitor plates to attract the dielectric into the plates. [1]
(Work has to be done to pull out the dielectric, to restored back the energy stored in the capacitor)

(6)(a)(i) The wavelength of sound sent out by siren = c/f

The speed of sound relative to the observer = u + c

Frequency heard by observer = (u+c)/(c/f) = (u+c)f/c [2]

(ii)As the motorcyclist if far away from the siren, its velocity is almost parallel to the velocity of sound and add on to it. So the observed frequency is higher than f by (c+u)f/c [1]

As the motorcyclist approaches the siren, the component of velocity in the direction of the wave becomes smaller $ucos(\theta), where\ \theta$ is the angle between the original direction and the current position. So the frequency is now (c+ $ucos(\theta)f/c$, which decreases as θ increase [1]

As θ becomes 90, the observed frequency drops to f. [1]

As θ increase beyond 90, the relative speed decrease to c-| $ucos(\theta)$|, so the frequency decrease till (c-u)f/c [1]

(b) In unit time, if the source is moving with a speed v_s, the f waves occupy a distance of ($c - v_s$), so the wavelength is now shorter in value, given by ($c - v_s$)/f. [1]

The speed of sound remains the same.

So the observed frequency = c/[($c - v_s$)/f] or $\frac{c}{c-v_s} f$ [1]

(c) Answer: $f_o = \frac{c+u}{c-v_s}$f

(d) The velocity of air relative to ground = v_w

Velocity of sound sound relative to air = c
Therefore velocity of sound wave relative to ground = v_w+ c [1]
So have to replace c by v_w+ c in the expression in part (c) [1]

(e) $$\frac{\Delta f}{f} = \frac{f_o - f}{f} = \frac{\left[\left(\frac{c}{c-v_s}\right)f - f\right]}{f} = \frac{1}{1-\frac{v_s}{c}} - 1 = \left(1 + \frac{v_s}{c} + \textit{higher terms}\right) - 1 = \frac{v_s}{c}$$

[1]

(ii) $$\frac{\delta f}{f} = \frac{v}{c} = \frac{2\pi R}{Tc}$$ [1]

Or $f = \frac{2\pi R}{Tc} f$.

The side moving towards Earth will have f+ δf, and the side moving away from the Earth has $f - \delta f$ [1]

(iii) Volume = $\frac{4\pi R^3}{3} = \frac{2.0 \; x \; 10^{30}}{1.4 \; x \; 10^3}$, R = 6.99 x 10^8 [1]

δf/f = 6.91 x 10^{-6} [1]

(iv) Line width = $2\Delta\lambda$, $\frac{2\Delta\lambda}{\lambda} = \frac{2\Delta f}{f} = \frac{2v}{c} = 8 \; x \; 10^{-5}$, [1]

v = 12000 m/s = $\sqrt{\frac{3kT}{m}}$, T = 5800 K [1]

7(a) Take pivot at contact point: $\frac{a_{cm}}{a} = \frac{2r}{3r}$, $a_{cm} = 2a/3$ [1]

(b) Take contact as pivot, $\alpha = \frac{a_{cm}}{2r} = \frac{\frac{2a}{3}}{2r} = \frac{a}{3r}$ = 13.3a [2]

(if take CM as pivot, $\alpha = \frac{a - \frac{2a}{3}}{r} = \frac{a}{3r}$)

(c) 2Mg -T = 2Ma force equation for M [1]

T + f – Mg sin(30) = M(2a/3) force equation for yoyo [1]

(d) Net torque about centre of mass = I x angular acceleration [1]

Tr – 2fr = $Mr^2(a/(3r))$ [1]

(e) Solving 3 equations

T =16 Mg/23 = 0.696 Mg

a = 15g/23 = 0.652 g

f = 51Mg/46 = 0.239 Mg [3]

(f) f = $\mu mgcos(30)$ [1]

$\mu = \frac{11}{23\sqrt{3}}$ = 0.276 [1]

(g) $\alpha = \frac{a}{3r} = \frac{0.652g}{3r} = 85.3$ [1]

$\theta = \frac{1}{2}\alpha t^2 = 10.7\ rad$ [1]

(h) (i) Translational KE + rotational KE = mgh

Consider centre of mass as reference: v = 2x2.5/3, $\omega = 2.5/3r$

$\frac{1}{2}M\left(\frac{2x2.5}{3}\right)^2 + \frac{1}{2}Mr^2\left(\frac{2.5}{3r}\right)^2 = Mgh$ [1]

Solving gives h = 0.177 m [1]

(ii) Distance travelled by centre of mass along slope = 0.177/sin(30) = 0.354 m [1]

$S = \frac{v+u}{2}t = \frac{0+\frac{2x2.5}{3}}{2}t$, t = 0.425 s [1]

(iii) Linear retardation of centre of mass = $\frac{\left(\frac{2x2.5}{3}\right)^2}{2x0.354} = 3.92$ [1]

$\alpha = \frac{3.92}{2r} = \frac{3.92}{0.050} = 78.4\ rads^{-2}$ [1]

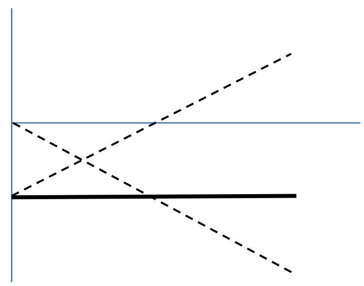
Or use $\alpha = \frac{\Delta\omega}{t} = \frac{0-\frac{2.5}{3x0.025}}{0.425}$ =78.4

8(a)(i)Treat the cylinder as a summation of I flowing out of page in the outer cylinder superimposed on a current flowing into the page in the hole.

The B-field due to outer cylinder inside the hole circulates in anticlockwise with magnitude that increases with distance from centre using ampere's law$B.(2\pi r) = \mu_o \pi r^2 J\ or\ B = \mu_0 Jr/2$ [1]

The B-field due to the inner cylinder circulates in clockwise manner from centre of small circle, increasing with magnitude from centre of small circle. [1]

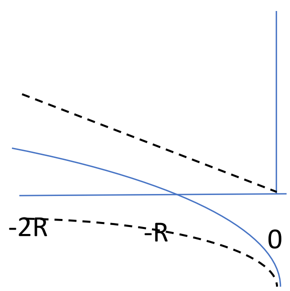
The vector sum of B is a constant value of $\mu_o JR/2$, pointing to the left. [1]



(ii)

B due to -J hole is 1/r, negative.

B due to +J large cylinder is proportional to r, positive.



(b)(i) Consider a circular path of radius r, the value of B uniform and tangential to the circular path [1]

Using Ampere's law: $B.2\pi r = \mu_0(NI)$ [1]

B = $\frac{\mu_0 NI}{2\pi r}$

$\Phi = \int d\Phi = \int BHdr$

$= \int \frac{\mu_0 NI}{2\pi r} Hdr$, int limit from a to b

$= \frac{\mu_0 NIH}{2\pi} \ln\left(\frac{b}{a}\right)$ [1]

(ii) $\frac{LdI}{dt} = \frac{d\Phi}{dt}\ or\ L = \frac{\Phi}{I}$ [1]

L = $\frac{\mu_0 NH}{2\pi} \ln\left(\frac{b}{a}\right)$ [1]

(c) (i) Torque = $\mu B sin(110)$ [1]

=12.9 Nm [1]

(II) Energy from 110 to 90 = 5.50x2.5cos(70)

Energy from 90 to 0 (lowest energy) = 5.50x 2.5
Total energy from 110 to 0 = 18.5 J [2]

(iii) Torque on magnetic moment when displace by θ from equilibrium = mBsinθ

Torque produces angular acceleration α, restoring to equilibrium point [1]

$mBsin\theta = -I\alpha = -\frac{Id^2\theta}{dt^2}$ [1]

$$\frac{d^2\theta}{dt^2} = \frac{-mBsin\theta}{I}$$

For small angle $\frac{d^2\theta}{dt^2} = \frac{-mB\theta}{I}$, which is simple harmonic in angular displacement. [1]

Angular frequency $\omega^2 = \frac{mB}{I}$, [1]

f=$2\pi/\sqrt{\frac{mB}{I}}$ = 0.656 Hz [1]

Q9 (a) Charge induced on the inner surface = -Q [1]

(b) (i) For radius r, charge enclosed = $\frac{\frac{4\pi}{3}r^3}{\frac{4\pi}{3}q^3}Q = \frac{r^3}{R^3}Q$ [1]

Gausss law: $4\pi r^2 E = \left(\frac{r^3}{q^3}\right) Q/\epsilon_0$ [1]

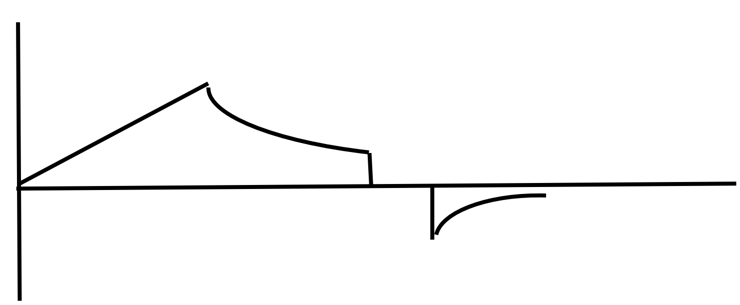
E = $\frac{rQ}{4\pi\epsilon_0 q^3}$ [1]

(ii) $E = \frac{Q}{4\pi\epsilon_0 r^2}$

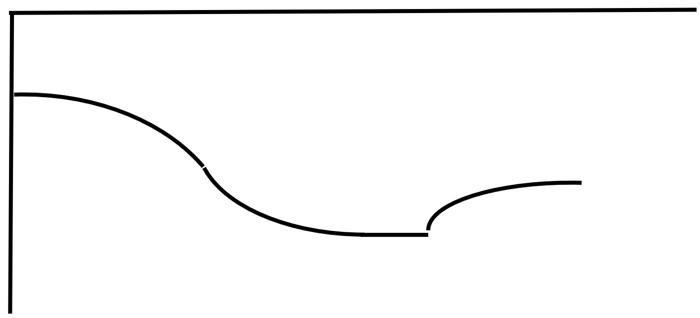
(iii) E = 0

(iv) Net charge = -Q, $E = \frac{-Q}{4\pi\epsilon_0 r^2}$

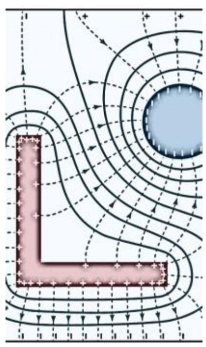
(c)(i)



(ii)



(d)



Correct arrow direction for E-lines , potential lines perpendicular to E [1]

E-lines repel at corner of inner edge. [1]

E-lines closer at sharp edge of rectangle. [1]

(e) Electric field is proportional to radial distance r from centre,

$$E = \frac{Q_{inside}}{4\pi\epsilon_o r^2} = \frac{\left(\frac{\frac{4}{3}\pi r^3 Q}{\frac{4\pi R^3}{3}}\right)}{4\pi r^2 \epsilon} = (\frac{Q}{(4\pi R^3\epsilon_o)}r \qquad [1]$$

Force F = $qE = \frac{qQr}{4\pi\epsilon_o R^3} = \frac{-md^2r}{dt^2}$

Acceleration $\frac{d^2r}{dt^2} = -(\frac{qQ}{4\pi\epsilon_0 mR^3})r$ is proportional to r, opposite in direction. [1]

Motion is simple harmonic about centre of sphere. [1]

Angular frequency of SHM is given by $\sqrt{\frac{qQ}{4\pi\epsilon_0 mR^3}}$ or period of oscillation is $\sqrt{(16\pi^3\epsilon_o mR^3/qQ)}$

and amplitude of oscillation is a (radius of sphere). [1]