NAME: ______________________ (   ) CLASS: SEC 4E (   )

HOUGANG SECONDARY SCHOOL

MID-YEAR EXAMINATION 2021

ADDITIONAL MATHEMATICS 4049/02

SECONDARY FOUR EXPRESS

11 May 2021, Tuesday **2 hours 15 minutes**

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Instructions to students:

- Write your name, class and register number in the spaces at the top of this page.
- Write in dark blue or black pen on spaces provided.
- You may use a HB pencil for any diagrams or graphs.
- Do not use staples, paper clips, glue or correction fluid.
- Answer **all** the questions in this paper.
- Give non-exact numerical answers correct to 3 significant figures, or one decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
- The use of an approved scientific calculator is expected, where appropriate.
- You are reminded of the need for clear presentation in your answers.

Information for pupils

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total mark for this paper is 90.

Calculator Model: ______________________

$\frac{\quad}{90}$

This question paper consists of 19 printed pages (including this cover page).

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+....+\binom{n}{r}a^{n-r}b^r+....+b^n$$

where n is appositive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)..........(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\cos ec^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 Express $\dfrac{3x^3+4x-2}{x^3+x}$ in partial fractions. [6]

2(a) Solve the equation $\sqrt{3-4x}+2x=0$ [4]

2(b) The lengths of the two shorter sides of a right-angled triangle are $\left(\sqrt{3}+\sqrt{5}\right)$ cm and $\left(\sqrt{48}-\sqrt{20}\right)$cm respectively. [3]

Find the area of the triangle in the form $\left(a+b\sqrt{15}\right)\text{cm}^2$.

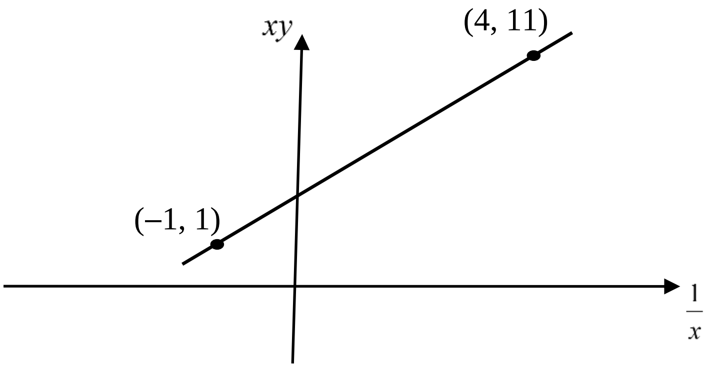
3(a) Given that $\frac{d}{dx}\left[x\sqrt{x+2}\right]=\frac{ax+b}{2\sqrt{x+2}}$, find the values of a and b. [4]

3(b) **(i)** The equation of a curve is $y = \dfrac{2x}{1-x^2}$. Find an expression for $\dfrac{dy}{dx}$. [3]

3(b) **(ii)** Using the result in **(i)** above, explain, with working, why the curve $y = \dfrac{2x}{1-x^2}$ has no turning points. [2]

4(a) The diagram shows the straight line graph obtained by plotting xy against $\frac{1}{x}$.

The line passes through the points $(-1,1)$ and $(4,11)$.



Express y in terms of x. [4]

4(b) An experiment on the discharge of an electrical circuit was conducted.
It is known that V volts, the voltage of the electrical circuit and t seconds, the elapsed time since the removal of the power source, are related by the equation $V = Ae^{kt}$, where A and k are constants.
The table below shows corresponding values of t and V.

t	1.0	1.5	2.0	2.5	3.0
V	2.80	2.20	1.35	1.39	1.10

(i) On the grid given in page 9, plot a straight line to illustrate this data. [5]
Use the graph to estimate the values of A and k.

(ii) Estimate the initial voltage of the electrical circuit [1]

(iii) It was subsequently discovered that one of the readings was wrongly recorded in the table. [2]
Using the graph, determine which value of V, in the table above, is the incorrect recording. Estimate a value of V to replace the incorrect recording of V.

5(a) **(i)** Find the range of values of p for which the equation $x^2+6x+p=2px-9$ has real and distinct roots. [4]

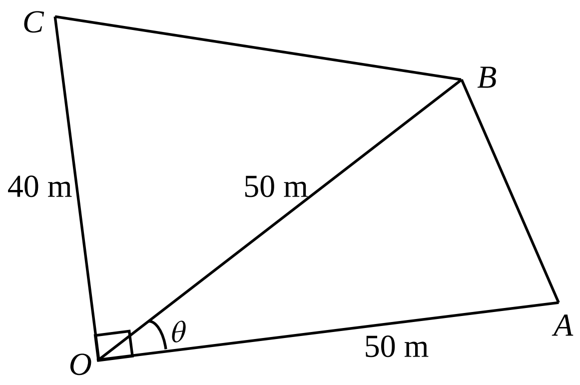
(ii) Using your answer in **(i)**, deduce whether the curve $y=(x+3)^2$ will meet the line $y=18x-9$. [2]

5(b) The curve with the equation $y = ax^2 - 3x + b$, where a and b are constants, lies completely above the x-axis.

(i) Find the conditions that must apply to a and b. [3]

(ii) Give an example of possible values of a and b which satisfy the conditions in part **(i)**. [2]

6 The figure shows two triangular plots of land OAB and OBC. It is given that $OA = OB = 50$ m , $OC = 40$ m, angle $AOC = 90^o$ and angle $AOB = \theta$.



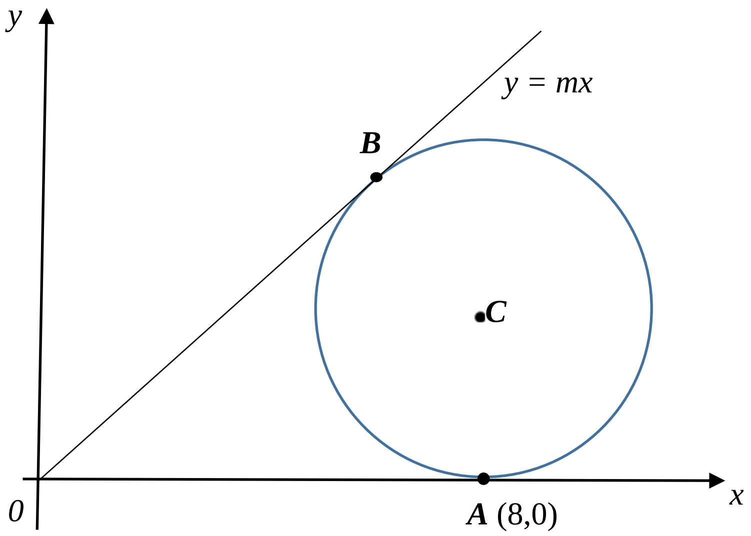
(a) Show that the sum of the areas of the two plots of land , A m^2 , is given by $A = 1250\sin\theta + 1000\cos\theta$. [2]

(b) Express the area, A in the form $R\sin\left(\theta+\alpha\right)$ and evaluate R and α where $R > 0$ and $0^o < \alpha < 90^o$. [3]

(c) Find the value of θ when $A = 1200$ m^2. [3]

(d) Is it possible to find a value of θ for A to be 1800 m^2? Give a reason. [1]

7 The diagram shows a circle with centre *C* and radius 4 units. The circle touches the positive *x*-axis at point *A* with coordinates (8, 0).



(a) Write down the coordinates of the centre *C*, and hence show that the equation of the circle is $x^2 + y^2 - 16x - 8y + 64 = 0$. [3]

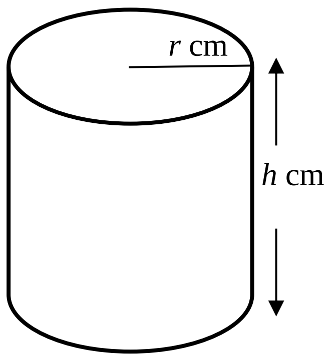
(b) A student claims that it is possible to draw a circle passing through the points *O*, *A*, *B* and *C*. Do you agree? Give a reason. [2]

(c) **(i)** A line $y = mx$, where m is a positive constant, passes through the origin O and touches the circle at B. [5]

Show that $m = \frac{4}{3}$.

(ii) Hence find the equation of the normal to the circle at point B. [3]

8 A manufacturer makes closed right cylindrical cans with a volume of 320 cm^3 each. The material for the top and bottom of the cylinder costs 3 cents per 10 cm^2 while the sides of the container costs 2 cent per 10 cm^2.
The diagram shows a cylindrical can with radius r cm and height h cm.



(a) Show that $h = \dfrac{320}{\pi r^2}$. [1]

(b) Assuming there is no wastage of materials during production, show that the cost of producing a can, P (in cents), can be expressed as [3]

$$P = 0.6\pi r^2 + \frac{128}{r}.$$

(c) Find the dimensions of each cylindrical can the manufacturer must use so that the cost of production is a minimum. Give your answers to 1 decimal place. [6]

(d) Hence, write down the minimum cost of producing one can, to the nearest cent. [1]

9(a) Use the substitution $u = 3^x$ to solve the equation $2\left(3^x\right) + 3^{-x} = 3$. [4]

(b) Solve the equation $\log_5 50 + 4\log_{25}(x-1) = 2$. [5]

(c) **(i)** Sketch the graph of $y = \ln x$ for $x > 0$, indicating the x-intercept. [1]

(ii) Amy wishes to find the number of solutions to the equation $x^2 = e^{1-x}$ by inserting a suitable line onto the sketch of $y = \ln x$ in **(i).**
Find the equation of the line to be drawn onto the sketch. [2]

---End of Paper---