**Sec 4E A Math PRELIM Examinations 2021 P2 Marking Scheme**

| Qn | Solutions |
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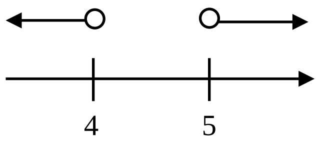
| 1 | $k^2+4k+4>13k-16$ [M1]<br>$k^2-9k+20>0$<br>$(k-4)(k-5)>0$ [M1]<br>$k<4,\ k>5$ [M1]<br><br>[A1] |
| 2(a) | $\frac{dy}{dx}=3(\frac{5}{6})\left(\frac{x}{4}+a\right)^{-\frac{1}{6}}(\frac{1}{4})$ [M1]<br>Gradient of normal = $-\frac{4}{5}$<br>Gradient of tangent = $\frac{5}{4}$ [M1]<br>$\frac{5}{4}=(\frac{15}{24})\left(\frac{0.5}{4}+a\right)^{-\frac{1}{6}}$ [M1]<br>$a=\ -\frac{7}{64}$ [A1] |
| 2(b) | $When\ x=0.5, y=3\left(\frac{0.5}{4}-\frac{7}{24}\right)^{\frac{5}{6}}=\ \frac{3}{32}$ [M1]<br>$y-\frac{3}{32}=\frac{5}{4}(x-0.5)$[M1]<br>$y=\frac{5}{4}x-\frac{17}{32}$ [A1] |
| 3(i) | Coefficient = $\binom{n}{r}(-3)^r$ [M1]<br>When $r$ = 2, $54=\binom{n}{2}(-3)^2\rightarrow 6=\frac{n(n-1)}{2}$ [M1]<br>$n=4$ [A1]<br>When $r$ = 1, $-6p=\binom{n}{1}(-3)^1\rightarrow 2p=n$ [M1] $\rightarrow p=2$ [A1]<br>When $r$ = 3, $q=\binom{4}{3}(-3)^3=-108$ [A1] |
| 3(ii) | $=(1-12x+54x^2-108x^3)(1+2x)$<br>$=\cdots+108x^3-108x^3+\cdots$[M1]<br>= 0 [A1] |
| 4(i) | [Removed due to CLT]<br>$\measuredangle GFA=\ \measuredangle GFA\ (Tangent-chord\ theorem)$ [M1]<br>$=\ \measuredangle DFE\ (Alt\ angles)$ [M1]<br>$=\ \measuredangle EDF\ (Isosceles\ triangle)$ [A1] |

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| 4(ii) | $\measuredangle DEF = 180° - 2\measuredangle EDF\ (Isosceles\ triangle)$<br>$\measuredangle DGF = 180° - 2\measuredangle DEF\ (angles\ in\ opp\ segment)$ [M1]<br>$= 2\measuredangle EDF\ (Isosceles\ triangle)$<br><br>$\measuredangle AGF = 180° - \measuredangle DGF$<br>$= 180° - 2\measuredangle EDF$ [M1]<br><br>$\measuredangle GAF = 180° - \measuredangle AGF - \measuredangle GFA$<br>$\measuredangle DGF = 180° - (180° - 2\measuredangle EDF) - \measuredangle EDF$ [M1]<br>$= \measuredangle EDF = \measuredangle GFA$ [A1] |
| 5(i) | $\sin\theta = \frac{BD}{70}$<br>$70\sin\theta = BD$ [M1]<br>$\cos\theta = \frac{BE}{30}$<br>$30\cos\theta = BE$ [M1]<br>$h = 70\sin\theta - 30\cos\theta$ [A1] |
| 5(ii) | $R = \sqrt{70^2 + 30^2}$ [M1]<br>= 76.2 (3sf)<br>$\propto = \tan^{-1}\left(\frac{30}{70}\right)$ [M1]<br>$= 23.2°$<br>$h = 76.2\sin(\theta - 23.2°)$ [A1] |
| 5(iii) | $20 = 76.2(\sin\theta - 23.2°)$<br>$\propto = \sin^{-1}\left(\frac{20}{76.15773}\right)$ [M1]<br>$= 15.22515°$<br>$\theta - 23.1985° = 15.22515°$<br>$\theta = 34.42365° = 34.4°$ [A1] |
| 6(i) | See attached<br>B1 – ln y values<br>B1 – axis/scale<br>B1 – smooth curve |
| 6(ii) | $Gradient = \frac{4.15-1.65}{8-2}$ [M1]<br>$\ln b = \frac{5}{12}$<br>$b = e^{\frac{5}{12}}$ [M1]<br>$y - intercept = 0.8$<br>$\ln a = 0.8$<br>$a = e^{0.8}$ [M1]<br><br>$y = (e^{0.8})(e^{\frac{5}{12}x})$ [A1] |
| 6(iii) | See attached<br>B1 – straight line<br>$x$ = 2.8 [B1] |
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| 7(a) | $x(5x-12)=2x-c$ [M1]<br>$5x^2-14x+c=0$<br>$(-14)^2-4(5)(c)<0$ [M1]<br>$9.8<c$ [A1] |
| 7(b) | $x^2-4x+\left(-\frac{4}{2}\right)^2-\left(-\frac{4}{2}\right)^2+7$ [M1]<br>$=(x-2)^2+3$ [M1]<br>Turning point = (2 , 3) [A1]<br>$minimum\ value=3$ |
| 7(c) | $2x^2-6x-4+2x=12$ [M1]<br>$(x-4)(x+2)=0$<br>$x=4, y=4$ [A1]<br>$x=-2, y=16$ [A1] |
| 8(a) | $\log_2 x^2-3\left(\frac{\log_2 4}{\log_2 x}\right)=\frac{\log_2 2}{\log_2 4}\times\frac{\log_2 16}{\log_2 4}$ [M1]<br>Sub $\log_2 x=y$<br>$2y-\frac{6}{y}=1$ [M1]<br>$2y^2-y-6=0$ [M1]<br>$y=-\frac{3}{2}, y=2$<br>$\log_2 x=-\frac{3}{2}$<br>$x=2^{-\frac{3}{2}}$ [A1]<br>$\log_2 x=2$<br>$x=2^2=4$ [A1] |
| 8(b) | $2^{6(x-1)}\times 2^{3x}=2^{4\left(\frac{6x}{3}\right)}$ [M1]<br>$2^{6x-6+3x}=2^{8x}$ [M1]<br>$9x-6=8x$ [M1]<br>$x=6$ [A1] |
| 9(i) | Gradient of *BD* = 1 [M1]<br>Gradient of *AC* = – 1 [M1]<br>$y=-x+1$ [A1] |
| 9(ii) | *C* (5 – 3, 4 – 5) [M1]<br>*C* (2, –1) [M1]<br>Area = $\frac{1}{2}\begin{vmatrix}0 & -3 & 2 & 5 & 0\\ 1 & -4 & -1 & 4 & 1\end{vmatrix}$ [M1]<br>= 16 units$^2$ [A1] |
| 9(iii) | Gradient of *AB* = $\frac{5}{3}$<br>Gradient of *p* = $\frac{5}{3}$ [M1]<br>Accept answer with similar meaning to “same gradient”, “parallel lines”, “does not intersect” [A1] |

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| 10(a) | $\frac{dy}{dx} = (x)\big(2(cosx)(-sinx)\big) + (cos^2x)(1)$ [M2]<br>$= cos^2x - 2xsinxcosx$ [A1] |
| 10(b) | $xcos^2x + c = \int cos^2x\,dx - \int xsin2x\ \ dx$ [M1]<br>$= \int \frac{1+cos2x}{2}dx - \int xsin2x\ \ dx$ [M1]<br>$xcos^2x + C_1 = \frac{1}{2}x + \frac{sin2x}{4} + C_2 - \int xsin2x\ \ dx$ [M2]<br>$\int xsin2x\ \ dx\ = \frac{1}{2}x + \frac{sin2x}{4} - xcos^2x + C$ [A1] |
| 10(c) | $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} xsin2x\,dx\ = [\frac{1}{2}x + \frac{sin2x}{4} - xcos^2x]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ [M1]<br>$= \frac{\pi}{4} - 0.47829$ [M1]<br>$= 0.307\,rad$ [A1] |
| 11(i) | $Base\ area =\ (60 - x)(45 - 2x)$ [M1]<br>Volume = $(2700 - 165x + 2x^2)(x)$ [M1]<br>$= (2x^3 - 165x^2 + 2700x)$ [A1] |
| 11(ii) | $\frac{dV}{dx} = 6x^2 - 330x + 2700$ [M1]<br>$0 =\ x^2 - 55x + 450$ [M1]<br>$(x - 45)(x - 10) = 0$ [M1]<br>$x = 45, x = 10$ [A1]<br>(rejected) |
| 11(iii) | $\frac{d^2V}{dx^2} = 12x - 330$ [M1]<br>$=\ 12(10) - 330 = -210 < 0$ [M1]<br>Stationary value is maximum, best possible design [A1] |