SPRINGFIELD SECONDARY SCHOOL
Preliminary Examination 2021

STUDENT NAME							
CLASS	**S**		**-**		INDEX NUMBER		

ADDITIONAL MATHEMATICS — 4049/01
Secondary 4 Express

25 Aug 2021
2 hours 15 minutes

Candidates answer on the question paper
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Total	**/90**

Do not turn over this question paper until you are told to do so.

This question paper consists of 16 printed pages.

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations. [5]

$$\frac{2}{x} + y = 9$$

$$x + \frac{5}{y} = \frac{3}{2}$$

2 The equation of a curve is $y = -1 - 2\sin 2x$.

(a) State the minimum and maximum values of y. [2]

(b) Sketch the graph of $y = -1 - 2\sin 2x$ for $0° \leq x \leq 360°$. [3]

3 **(a)** Explain why there are no odd powers of x in the expansion of $\left(kx - \frac{2}{x}\right)^{10}$, where k is a constant. [3]

(b) Given $k = 3$, find the coefficient of x^2 in the expansion of $\left(kx - \frac{2}{x}\right)^{10}$. [3]

4 The function f is defined by $\mathrm{f}(x) = \dfrac{x+1}{x^2+3}$.

(a) Determine the range of values of x for which f is a decreasing function. [4]

(b) A point T moves along the curve $y = \mathrm{f}(x)$ in a way that the y-coordinate of T is decreasing at a rate of 0.1 units per second. Find the rate of increase of the x-coordinate of T when $x = 2$. [2]

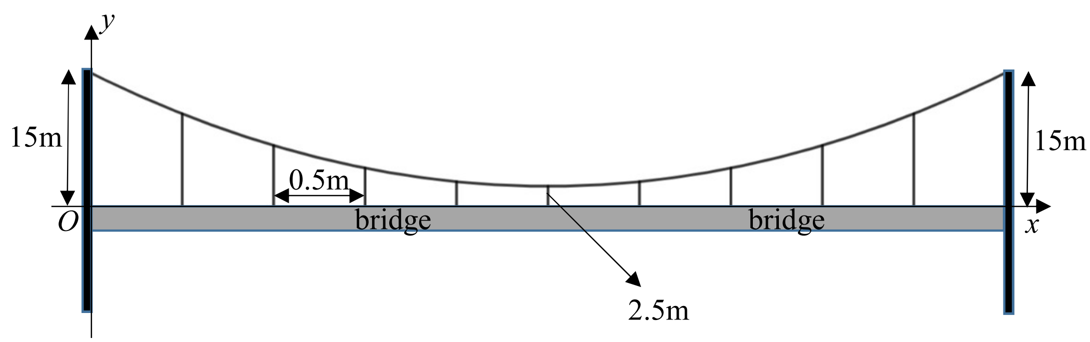
5 The population, P of a certain micro-organism, present at t hours after initially being observed is given by the formula $P = 160 + 220e^{kt}$, where k is a constant. It took 4 hours to reach a population of 620.

(a) Calculate the population of the micro-organisms after 24 hours. [4]

(b) Explain why the formula is not suitable to calculate the long-term population of the micro-organisms. [2]

6 For a particular curve $\frac{d^2y}{dx^2} = \sin x - 3\cos 3x$. The curve passes through the point $P\left(\frac{\pi}{2}, 4\right)$ and the gradient of the curve at P is 4. Find the equation of the curve. [6]

7 A portion of a suspension bridge has a curved cable hanging between two pillars which can be modelled by a quadratic function. In the model, x m is the horizontal distance from the left pillar and y m is the height of the cable above the bridge. Vertical supporting wires are spread out in 10 equal intervals of 0.5 m apart. At its lowest point, the cable is situated 2.5 m above the bridge.



(a) Write down the co-ordinates of the lowest point. [1]

(b) Form a quadratic equation in the form $y = a(x+b)^2 + c$ to model this situation. Hence, find the value of a, b and c. [4]

8 Given that $\sin A = \frac{12}{13}$ and $\tan B = -\frac{4}{3}$, where A and B are in the same quadrant.

Evaluate

(a) $\tan(A - B)$, [3]

(b) $\sin 2B$, [2]

(c) $\cos(A + B)$. [2]

9 A rectangular sheet of metal is folded to form a shelter as shown in Figure A. The dimensions of the cross-section is shown in Figure B.
$ABDE$ is a rectangle with $AB = 2$ m and BCD is an isosceles triangle with $BC = CD = 4$ m. $\angle CBD = \angle CDB = \theta°$.

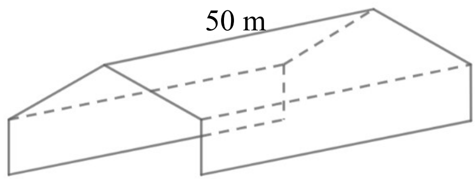


Figure A

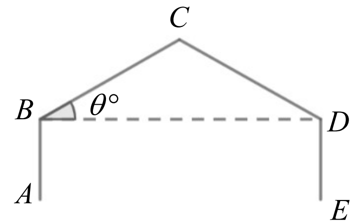


Figure B

(a) If the length of the shelter is 50 m, show that the internal volume of the shelter, V m^3, is given by $V = 400(\sin 2\theta + 2\cos\theta)$. [4]

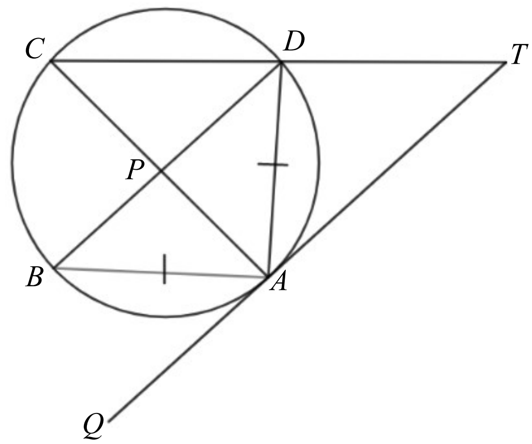
(b) Hence find the value of θ for which V is maximum and the maximum volume. [5]

10 The curve $y = 6x^2 + 1$ intersects another curve $y = \frac{k}{x} - 13x$ at 3 distinct points.
The x-coordinate of one of the intersection points is -2.

(a) Show that $k = 2$. [3]

(b) Hence, find the x-coordinate of the other 2 points of intersection. [5]

11 In the diagram, A, B, C and D lie on the circle. QT is the tangent to the circle at A. CT is a straight line that intersects the circle at D. Chords AC and BD intersect at P. Triangle ABD is an isosceles triangle with $AB = AD$.



Show that

(a) the line BD is parallel to line AT, [4]

(b) angle $PAD +$ angle $ATD = 180° - 2 \times$ angle PBA. [4]

12 **(a)** Prove the identity $\dfrac{1+\sin 2\theta+\cos 2\theta}{\cos\theta+\sin\theta}=2\cos\theta$. [4]

(b) Hence solve the equation $\dfrac{1+\sin 2\theta+\cos 2\theta}{\cos\theta+\sin\theta}=\sec\theta$ for $0<\theta<\pi$. [3]

13 A particle P is moving in a straight line from a fixed point O. The velocity, v m/s, is given by $v = 8 - \frac{1}{2}t^2$, where t is the time in seconds after passing O.

(a) Find the acceleration of P at $t = 6$. [2]

(b) Find the value of t when P is instantaneously at rest. [2]

(c) Find the time where the particle return back to point O. [4]

(d) Find the total distance travelled by the particle in the first 8 seconds. [4]

End of Paper