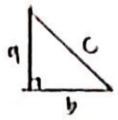
Pythagoras' Theorem and Trigonometry (AKA triangles)

→ Pythagoras' Theorem
- must be right-angled △
- $c^2 = a^2 + b^2$



→ Trigonometry
- $\sin\theta = \frac{o}{h}$
- $\cos\theta = \frac{a}{h}$
- $\tan\theta = \frac{o}{a}$

→ when you have either:
1. length of 2 sides
2. 1 angle and 1 length

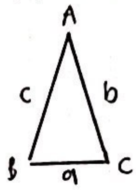
and wants to find unknown ∡ or length.

→ to find obtuse θ:
- $\sin(180° - \theta) = \sin\theta$
- $\cos(180° - \theta) = -\cos\theta$
- $\tan(180° - \theta) = -\tan\theta$

→ for ∡s that are not right-angled △, can use cosine rule or sine rule
- $a^2 = b^2 + c^2 - 2bc\cos A$
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $c^2 = a^2 + b^2 - 2ab\cos C$

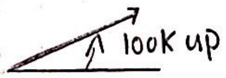
they are all the same

- Area of △ $= \underbrace{\frac{1}{2}ab\sin C \text{ or } \frac{1}{2}bc\sin A \text{ or } \frac{1}{2}ac\sin B}_{\text{all the same}}$

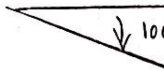


- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ → to find length
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ → to find angle

→ Angle of elevation & depression (use Pythagoras' Theorem or/and trigonometry to solve problems)
- Angle of elevation

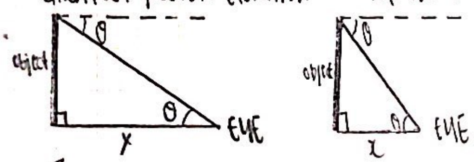


Angle of depression



→ a way to remember: people→sad, tend to look ↓

- GREATEST possible elevation or depression occurs when it has the SHORTEST distance between base of object and the eye

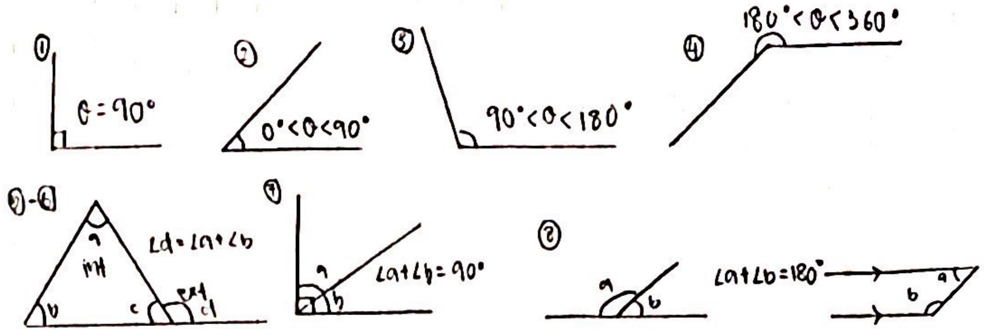


Shorter the distance of x → larger the ∡ of elevation/depression, θ.
*For this case, ∠ of elevation = ∠ of depression (alt. ∡)

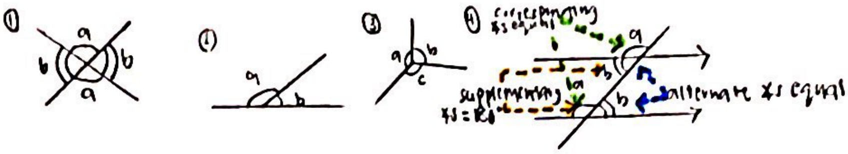
→ Bearings
- what to remember:
1. where is North direction pointing @? → not necessary be ↑N, could be ↓N
2. Calculate clockwise direction from north direction ⟳
3. write in 3 digits (e.g. 050°)
4. calculate from point of interest

A·N·G·L·E·S

→ Types of angles
1. Right angle
2. Acute angle
3. Obtuse angle
4. Reflex angle
5. Interior angle
6. exterior angle
7. complementary angle
8. supplementary angle



→ Properties of angles



1. vertically opposite angles are equal
2. Sum of adjacent ∡s on a straight line = 180° (supp. ∡s)
3. sum of adjacent ∡s at a point = 360°
4. for angles included in parallel lines (corresponding ∡s equal, supplementary ∡s = 180°, alternate ∡s equal)

→ Properties of angles in • circles
• polygons
• triangles
• ~~q~~ quadrilaterals

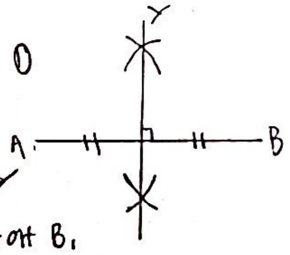
that can help (not necessary to memorise most)

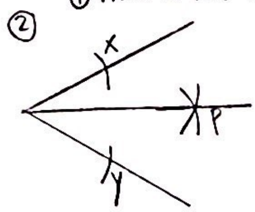
→ Polygons ←

# of sides	Name
3	Triangle
4	Square/Rectangle/Parallelogram/Quadrilateral/Rhombus/Kite/~~Trap~~ Trapezium
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
n	N-gon

FORMULAE for polygons:
- sum of interior $\angle = (n-2)\times 180°$
- sum of exterior $\angle = 360°$
- interior $\angle$ + exterior $\angle = 180°$
- each interior angle $= \frac{(n-2)\times 180°}{n}$ } for regular polygons
- each exterior angle $= \frac{360°}{n}$ } for regular polygons

→ Bisectors ←
1. Perpendicular bisector } types of bisectors
2. Angle bisector





① Put compass at A, mark arc above and below the line. ② using the same extension/radius of compass at B, mark arc above and below the line intersecting previous arcs. ③ Draw a line down joining the 2 intersections.

① compass at B, draw arcs to ~~cut~~ cut line AB at X and BC at Y. (same compass extension/radius for both) ② compass at X, draw arc between lines AB and BC. ③ compass at Y, draw another arc to previous arc at P. (same extension/radius as ②) ④ Draw a line to connect B and P

Mensuration

→ Polygons ←

→ Rectangle

- $P = 2(x+y)$
  $A = xy$

→ Square

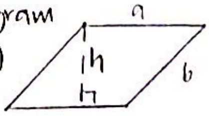
- $P = 4x$
  $A = x^2$

→ Triangle

- $P = a+b+c$
  $A = \frac{1}{2}ab\sin C$ or $\frac{1}{2}bh$

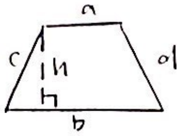
→ Parallelogram

- $P = 2(a+b)$
  $A = ah$



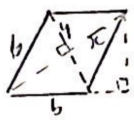
→ Trapezium

- $P = a+b+c+d$
  $A = \frac{1}{2}h(a+b)$



→ Rhombus

- $P = 4b$
  $A = bh$ or xy

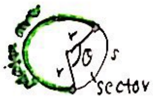


→ circle ←

- circumference of circle, $P = 2\pi r$
  $= \pi d$
  since $d = 2r$

- Area of circle, $A = \pi r^2$

→ sector ←

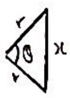


- Perimeter of sector, $P = s + 2r$
- arc $s = \frac{\theta}{360°} \times 2\pi r$ (where θ is in degrees)
- arc $s = \frac{\theta}{2\pi} \times 2\pi r = r\theta$ (where θ is in radians)
- Area of sector, $A = \frac{\theta}{360°} \times \pi r^2$ (θ is in degrees)
  $= \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2}r^2\theta$ (θ is in radians)

→ segment ←

- Perimeter of segment, $P = x + s$
  where (by using cosine rule),
  $x^2 = r^2 + r^2 - 2r^2\cos\theta$
- $x = \sqrt{2r^2 - 2r^2\cos\theta}$
- $s = \frac{\theta}{360°} \times 2\pi r$ (where θ is in degrees)
- $s = \frac{\theta}{2\pi} \times 2\pi r = r\theta$ (where θ is in radians)

- Area of segment, A = area of sector − area of triangle
  $= \left(\frac{\theta}{360°} \times \pi r^2\right) - \frac{1}{2}r^2\sin\theta$ (θ is in degrees)
  $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ (θ is in radians)



→ Regular / uniform solids ← (cuboid, cylinder, prism)

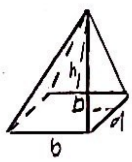
- volume = sum of all the cross-sectional areas
  = cross-sectional area × height

cross-sectional area
cuboid → rectangle
cylinder → circle
prism → triangle

→ Irregular / non-uniform solids ←

→ pyramid

- vol $= \frac{1}{3} \times$ base area × height
  $= \frac{1}{3} \times bd \times h$



For lengths

$\boxed{\text{To convert } m^2 \to cm^2}$

$1m = 100cm$

$1m^2 = 1m \times 1m$

$= 100cm \times 100cm$

$= 10000cm^2$

$\boxed{\text{To convert } cm^2 \to m^2}$

$1cm = 0.01m$

$1cm^2 = 1cm \times 1cm$

$= 0.01m \times 0.01m$

$= 0.0001m^2$

$m^3 \to cm^3$: × 1000 000
$cm^3 \to m^3$: ÷ 1000000

For circles

Angles can be measured in either degrees or radians, where $180° = \pi$ rad

$\boxed{\text{degree} \to \text{radian}}$:

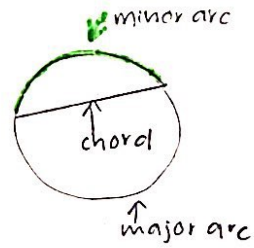
$180° = \pi$ rad

$1° = \frac{\pi}{180}$ rad

$\boxed{\text{radian} \to \text{degree}}$:

π rad $= 180°$

1 rad $= \frac{180}{\pi}$



→ next page

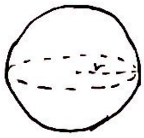
→ cone

- vol = $\frac{1}{3}$ x base area x height
  $= \frac{1}{3}\pi r^2 h$

- curved surface area $= \pi r l$
- Area $= \pi r l + \pi r^2$

→ sphere

- vol $= \frac{4}{3}\pi r^3$
- Area $= 4\pi r^2$



→ hemisphere

- vol $= \frac{2}{3}\pi r^3$
- Area $= 2\pi r^2 + \pi r^2 = 3\pi r^2$