Example 1

A spring, obeying Hooke's Law, has an unstretched length of 50 mm and a spring constant of 400 N m^{-1}.

What is the tension in the spring when its overall length is 70 mm?

Convert to base SI units

The extension of the spring is x = 70 – 50 = 20 mm = 0.020 m

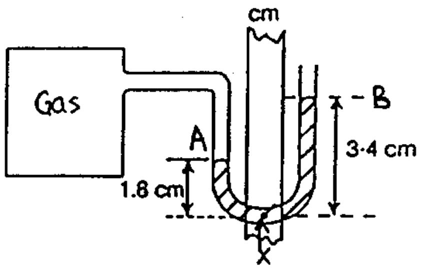
The tension in the spring is $T = k\,x$

$= (400)\,(0.020)$

$= 8.0$ N

Example 2

The diagram on the right shows a mercury manometer connected to a gas container. The atmospheric pressure is 760 mm Hg and the point X is at the bottom of the U-tube.



a) What is the pressure of the point X in mm Hg?

b) What is the pressure of the gas in mm Hg?

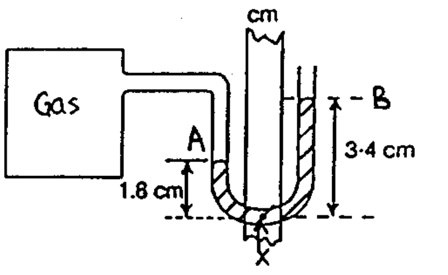
c) If the manometer is disco
what will be the vertical h
the point X in the right an

Pressure in mm Hg is defined as the pressure due to a column of so many mm of mercury

a) (pressure at point X) = (pressure due to fluid from B to X) + (atmospheric pressure)
= (34 mm Hg) + (760 mm Hg)
= 794 mm Hg

Example 2

The diagram on the right shows a mercury manometer connected to a gas container. The atmospheric pressure is 760 mm Hg and the point X is at the bottom of the U-tube.



a) What is the pressure of the point X in mm Hg?

b) What is the pressure of the gas in mm Hg?

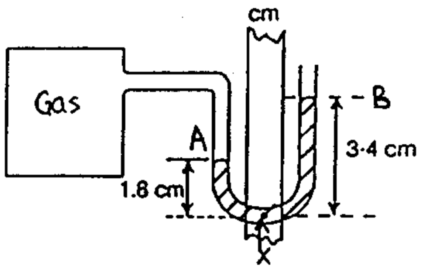
a) (pressure at point X) = 794 mm Hg

b) (pressure of the gas) + (pressure due to fluid from A to X)
= (pressure at point X)

(pressure of the gas) = (pressure at point X)
- (pressure due to fluid from A to X)
= (794 mm Hg) – (18 mm Hg)
= 776 mm Hg

Example 2

The diagram on the right shows a mercury manometer connected to a gas container. The atmospheric pressure is 760 mm Hg and the point X is at the bottom of the U-tube.



c) If the manometer is disconnected from the gas container, what will be the vertical heights of the mercury levels from the point X in the left and right limbs of the manometer?

c) If the manometer is disconnected from the gas container, both open ends of the tube will be at atmospheric pressure: the pressure at the top of each column will be the same.
Thus, the mercury will distribute so that it reaches the same height in both limbs.
The height will be (18 + 34) / 2 = 26 mm.

Example 3

Icebergs are often a danger to ships, since most of an iceberg's volume lies beneath the surface of the water. One can only see the "tip of the iceberg."

Find the percentage of a floating iceberg that is submerged.

(density of ice ρ_i = 0.917 g cm^{-3}, density of sea water ρ_w = 1.025 g cm^{-3})

The upthrust acting on the floating iceberg is equal to its weight.

From Archimedes' Principle, upthrust = weight of water displaced by the submerged part of the iceberg

$$U = W$$

$$m_w\, g = m_i\, g$$

$$m_w = m_i$$

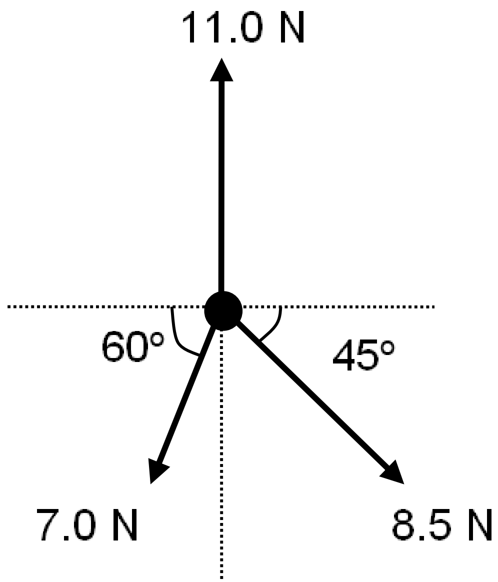
$$\rho_w\, V_w = \rho_i\, V_i$$

$$V_w / V_i = \rho_i / \rho_w = (0.917) / (1.025) = 0.895$$

Since the volume of the displaced water is 89.5% of the volume of the iceberg, 89.5% of the iceberg is submerged in the sea water.

Example 4

Four forces act on a particle such that it is in equilibrium. Three of the forces are shown in the diagram. Find the magnitude and direction of the unknown force *F* that maintains the equilibrium.



Thus, the horizontal component of the resultant of the three forces R_X is given by, (taking rightwards positive)

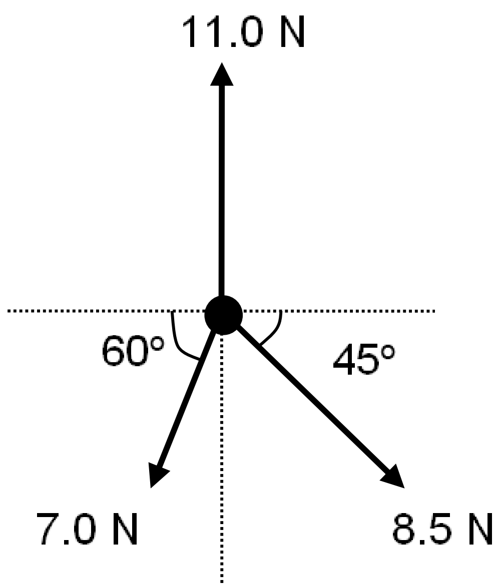
$R_X = (-7.0 \cos 60°) + (8.5 \cos 45°) = 2.51$ N

The vertical component of the resultant of the three forces R_Y is given by, (taking upwards as positive)

$R_Y = 11.0 - (7.0 \sin 60°) - (8.5 \sin 45°) = -1.07$ N

Example 4

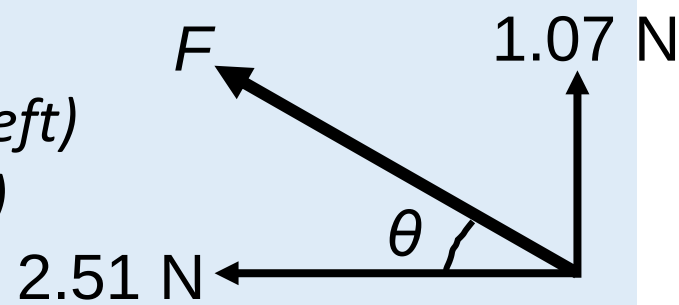
Four forces act on a particle such that it is in equilibrium. Three of the forces are shown in the diagram. Find the magnitude and direction of the unknown force *F* that maintains the equilibrium.



For the particle to be in equilibrium, the force *F* must be *equal in magnitude* and *opposite in direction* to the vector sum of the three forces.

$F_{\text{horizontal}}$ = - R_X = -2.51 N (i.e. 2.51 N *to the left)*

F_{vertical} = - R_Y = 1.07 N (i.e. 1.07 N *upwards)*



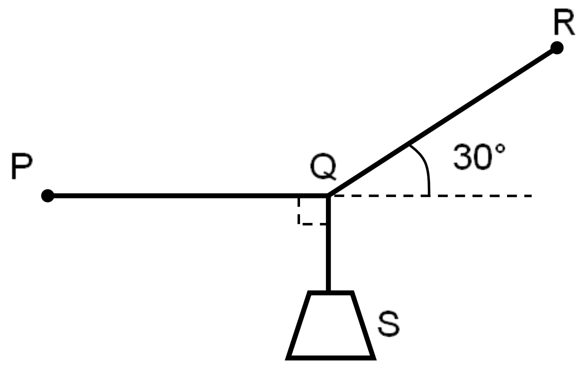
Using Pythagoras' theorem, the magnitude of *F* is 2.73 N.

tan θ = 1.07 / 2.51, so θ = 23°

Hence, *F* has a magnitude of 2.73 N and it is pointing at 67° anticlockwise from the force of 11.0 N.

Example 5

In the diagram on the right, a body S of weight *W* hangs vertically by a thread tied at Q to the string PQR. If the system is in equilibrium, what is the tension in section PQ?.



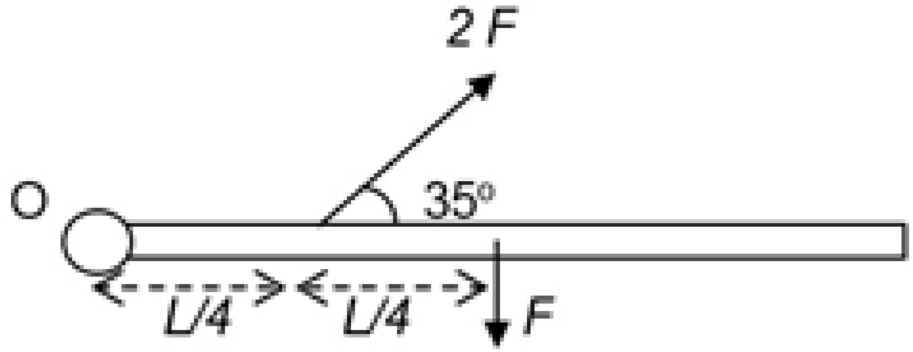
For the system to be in equilibrium, the *vertical* component of the tension in section QR must be equal to the tension in the section QS, which in turn must be equal to the weight *W*.

$$T_{\mathrm{QR,vertical}} = W$$
$$T_{\mathrm{QR}} \sin 30° = W$$
$$T_{\mathrm{QR}} = W / (\sin 30°) = 2.0W$$

For the system to be in equilibrium, the tension in section PQ must be equal to the *horizontal* component of the tension in section QR.

$$T_{\mathrm{PQ}} = T_{\mathrm{QR}} (\cos 30°) = 1.73W$$

Example 6



A uniform rod of weight F and length L is pivoted on one end O.
A force $2F$ is applied at a quarter of its length from O.
Find the magnitude of the moment about point O.

The moment about O due to the rod's weight is

$(F)\ (L/2) = 0.500FL$ (in the clockwise direction)

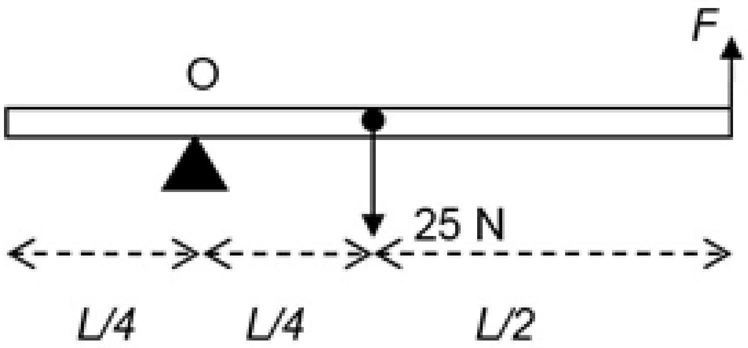
The moment about O due to the force $2F$ is

$(2F)\ (L/4)\ (\sin 35°) = 0.287FL$ (in the anticlockwise direction)

Hence, the net moment about O is

$(0.500FL) – (0.287FL) = 0.213FL$ (in the clockwise direction)

Example 7



A uniform rod of weight 25 N and length L is balanced on a knife edge at O. Find the unknown force F required to maintain rotational equilibrium.

By the principle of moments, the sum of the clockwise moments about any point must be equal to the sum of the anticlockwise moments about that same point.

Taking moments about O, the clockwise moment due to the weight

$$\tau_{\text{clockwise}} = (25\text{ N})\,(L/4)$$

Moments about O must be equal in magnitude

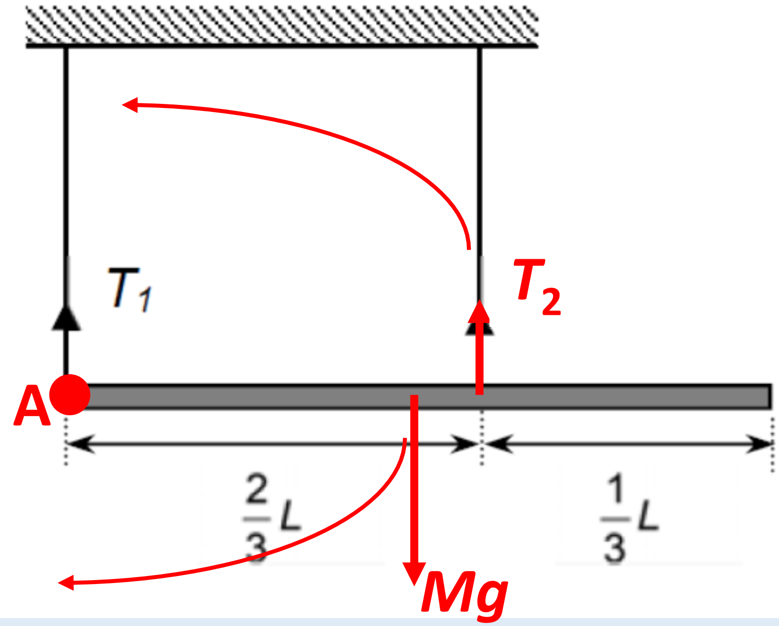
Hence, the anticlockwise moment about O due to F is

$$\tau_{\text{anticlockwise}} = (F)\,(3L/4) = (25\text{ N})\,(L/4)$$

Simplifying, we find $F = (25\text{ N}) / 3 = 8.33\text{ N}$

Example 8

A uniform beam of length L and mass M is supported by two vertical cords as shown in the diagram. What is the ratio T_1/T_2 of the tension in these cords?



We will be applying the principle of moments twice.

First, we take moments about point **A**.

By the principle of moments, the clockwise moment due to the weight Mg must be equal to the anticlockwise moment due to T_2.

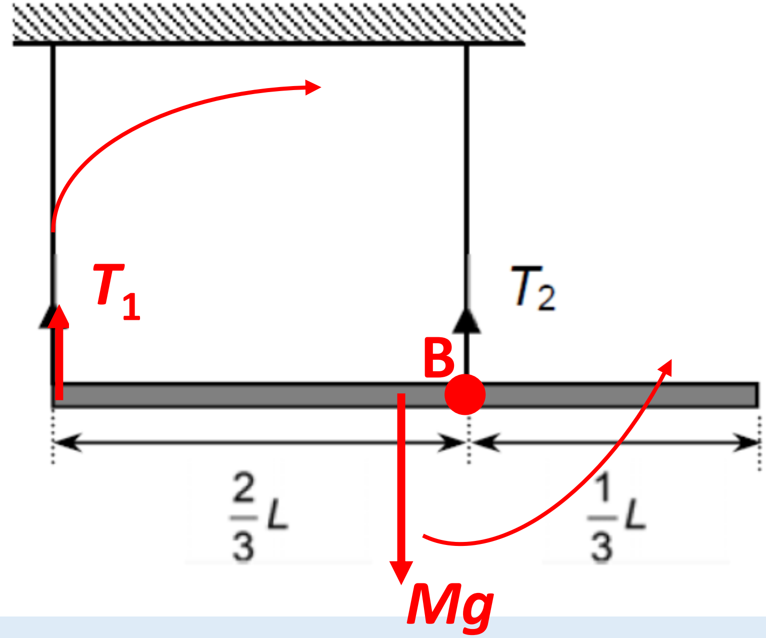
$$(Mg)(L/2) = (T_2)(2L/3)$$

$$T_2 = (Mg)(3/4) = 3Mg/4$$

The weight Mg acts at the centre, a distance $L/2$ away from A

Example 8

A uniform beam of length L and mass M is supported by two vertical cords as shown in the diagram. What is the ratio T_1/T_2 of the tension in these cords?



$T_2 = 3Mg/4$

Next, we take moments about point **B**.

By the principle of moments, the clockwise moment due to T_1 must now be equal to the anticlockwise moment due to the weight Mg.

$(T_1)\ (2L/3) = (Mg)\ (L/6)$

$T_1 = Mg/4$

Avoid leaving fractions in answers

Convert to decimals

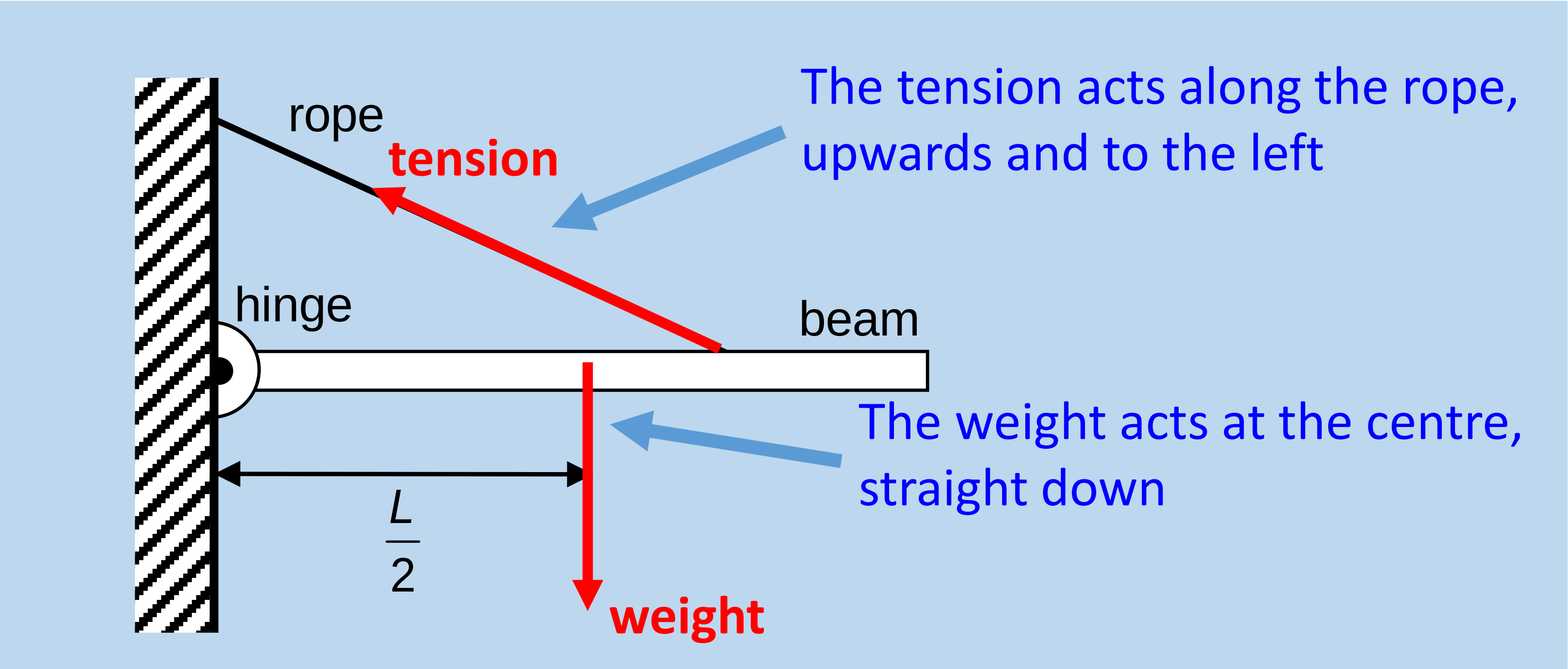
The weight Mg acts at the centre, a distance $L/6$ away from B

Combining, we find

$T_1/T_3 = 1/3 = 0.333.$

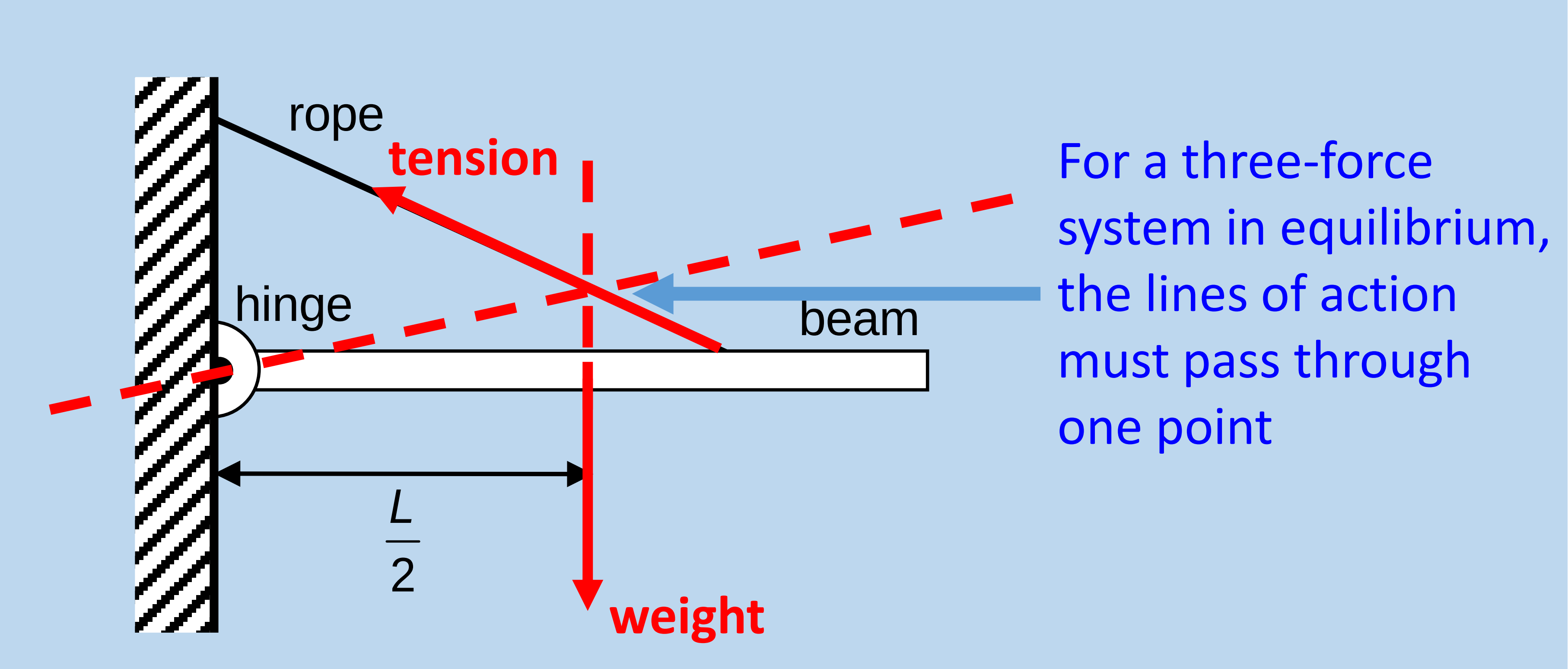
Example 9

A uniform beam balance of length L is hinged to a wall and is supported by a rope as shown. Show the direction of the force exerted by the hinge (attached to the wall) on the beam.



Example 9

A uniform beam balance of length *L* is hinged to a wall and is supported by a rope as shown. Show the direction of the force exerted by the hinge (attached to the wall) on the beam.



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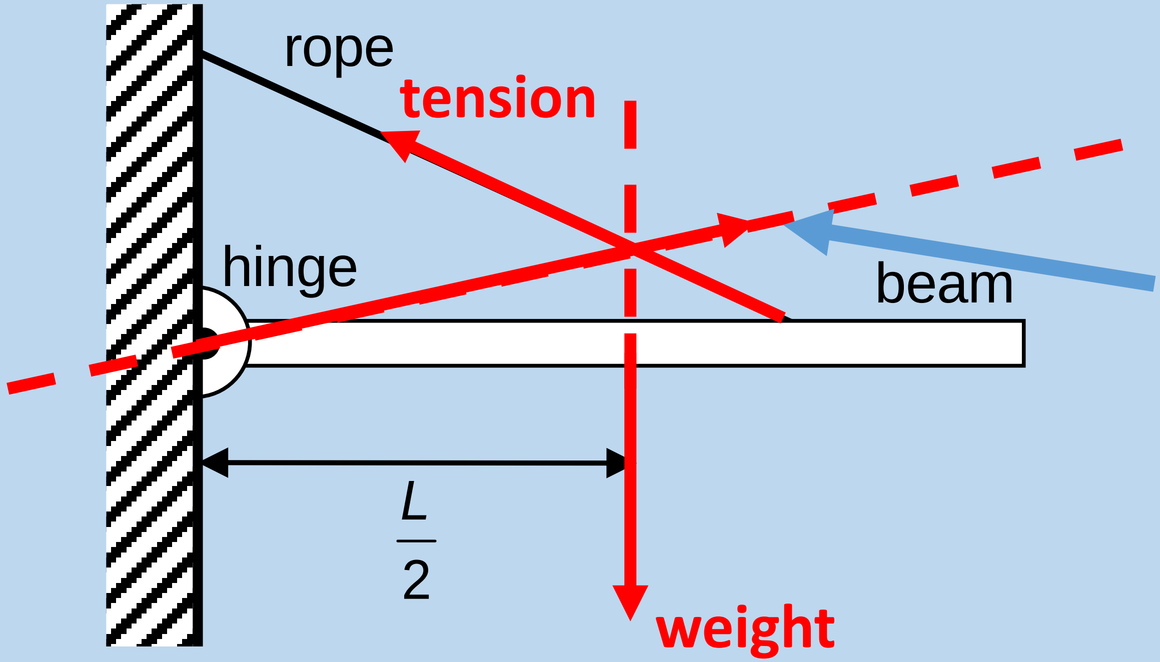
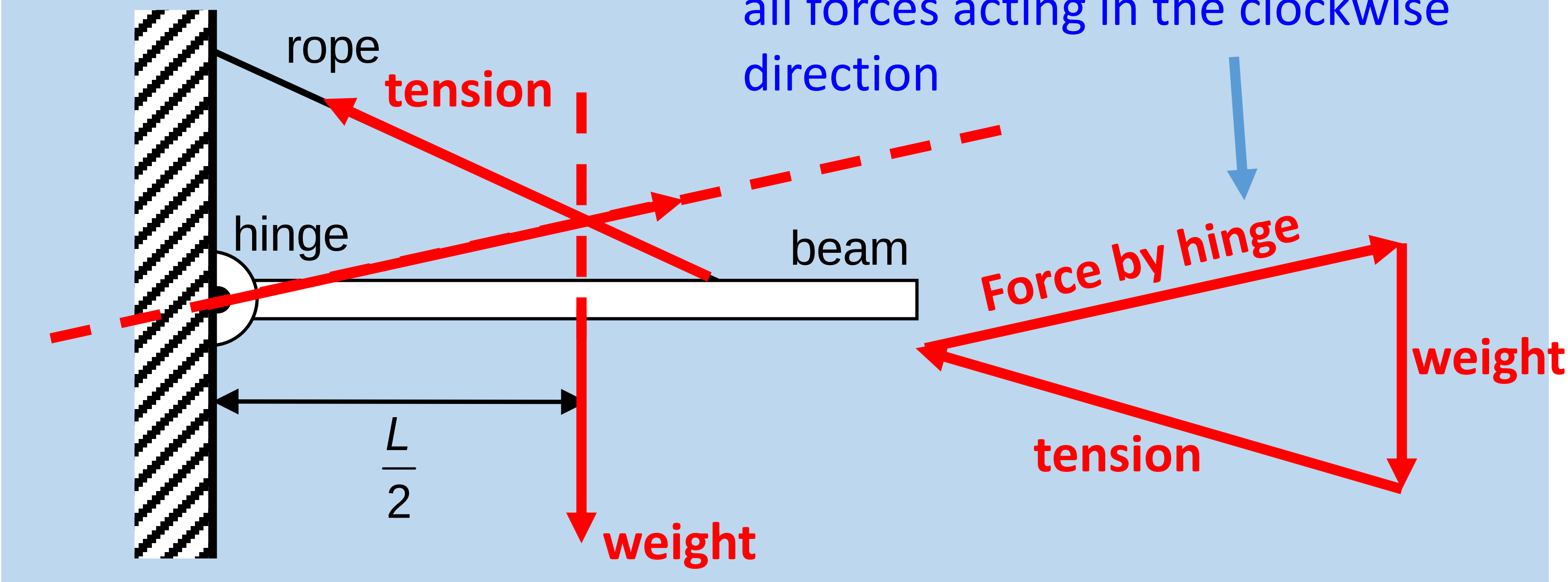


Since the net force must be zero, the force by the hinge must have a rightward component

Example 9

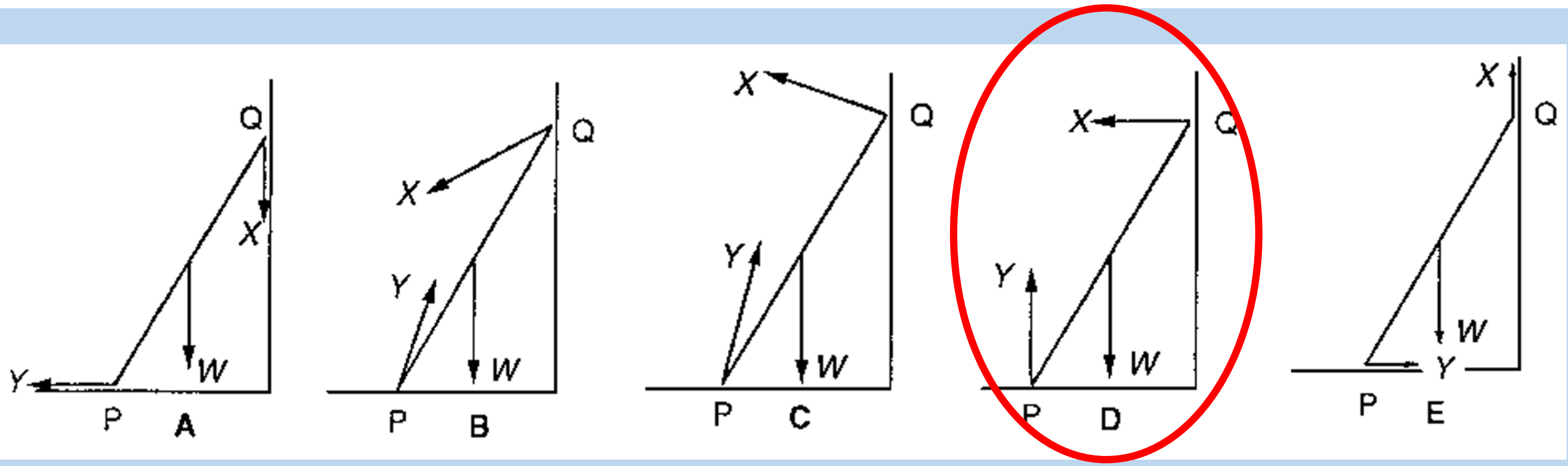
A uniform beam balance of length L is hinged to a wall and is supported by a rope as shown. Show the direction of the force exerted by the hinge (attached to the wall) on the beam.

A quick check : when joining vectors, head to tail, a closed Δ is formed with all forces acting in the clockwise direction



Example 10

A ladder PQ, resting on a rough floor and leaning against a rough wall, is on the point of slipping. It is of weight *W* and the contact forces exerted on the ladder by the wall and the floor are X and Y, respectively. Which one of the following diagrams correctly shows the directions of these forces?



Since the wall and the floor are rough, *X* and *Y* must have components due to friction *along* the wall and the floor. Option D is wrong.

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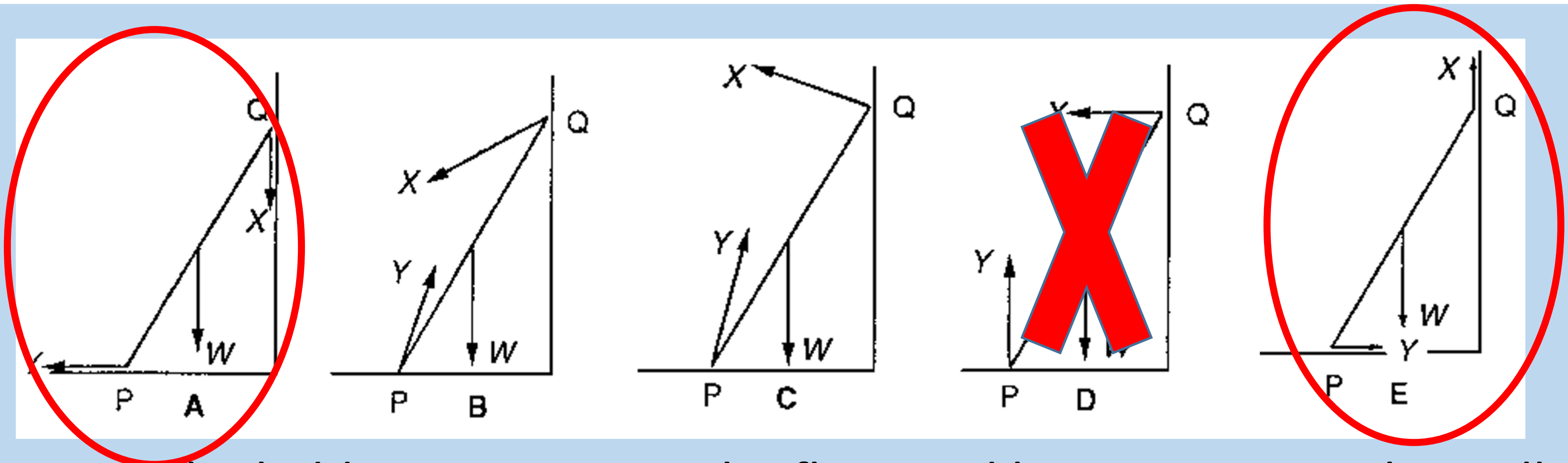
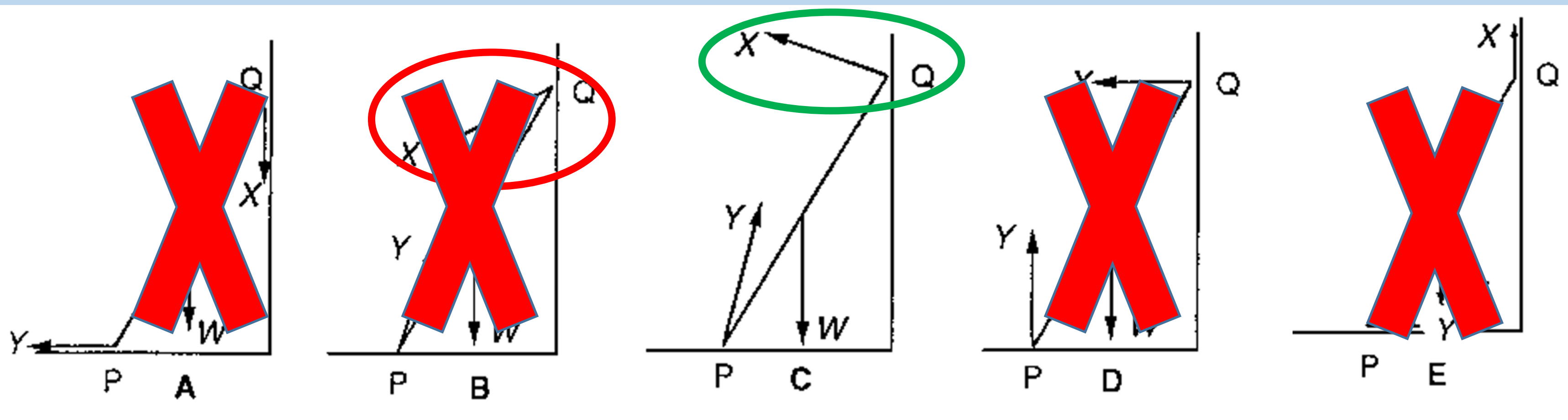


Since the ladder is resting on the floor and leaning against the wall, there must be normal contact forces from the floor and the wall. Options A and E are wrong.

Example 10

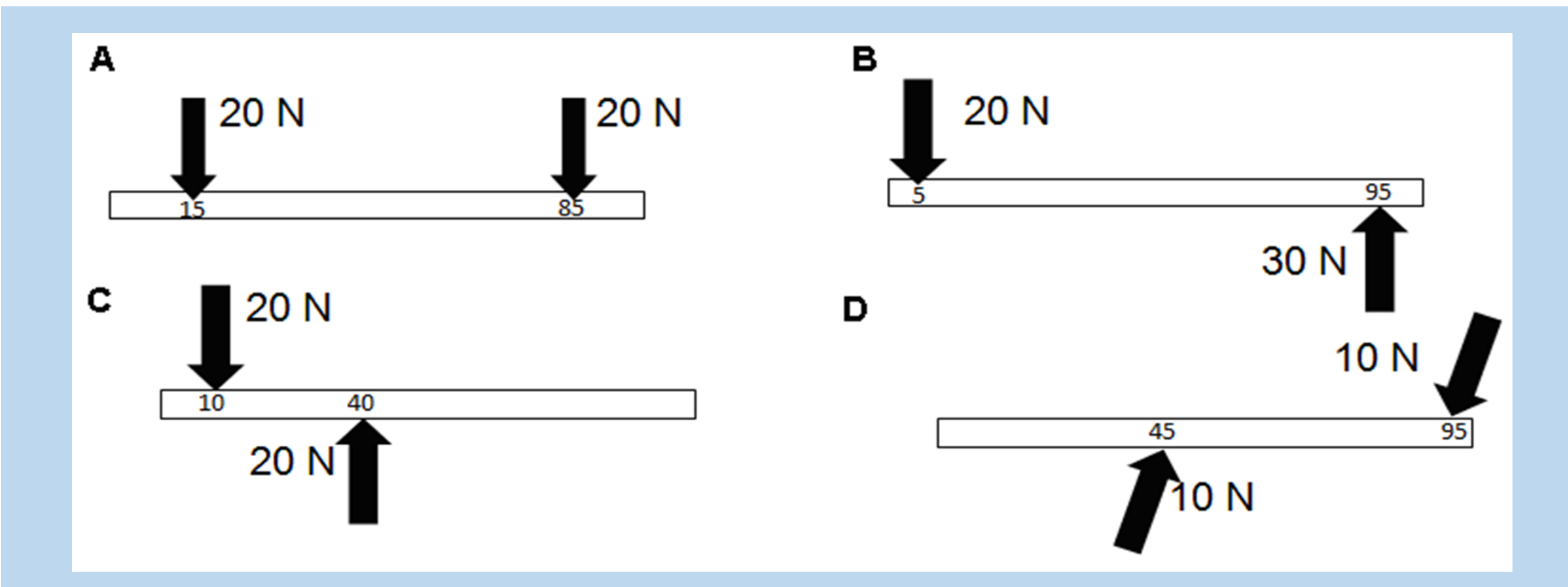
A ladder PQ, resting on a rough floor and leaning against a rough wall, is on the point of slipping. It is of weight *W* and the contact forces exerted on the ladder by the wall and the floor are X and Y, respectively. Which one of the following diagrams correctly shows the directions of these forces?



Since the ladder tends to slide down, there must be an *upward* frictional force at point Q.
Option C is correct.

Example 11

The following diagrams show forces acting on uniform metre rules. The rulers are lying on a horizontal smooth table top and shown as seen from above.
Which of the diagrams do not show a couple?
For the other diagrams, what is the torque of the couple?
Assume that for each option, the two forces are parallel.



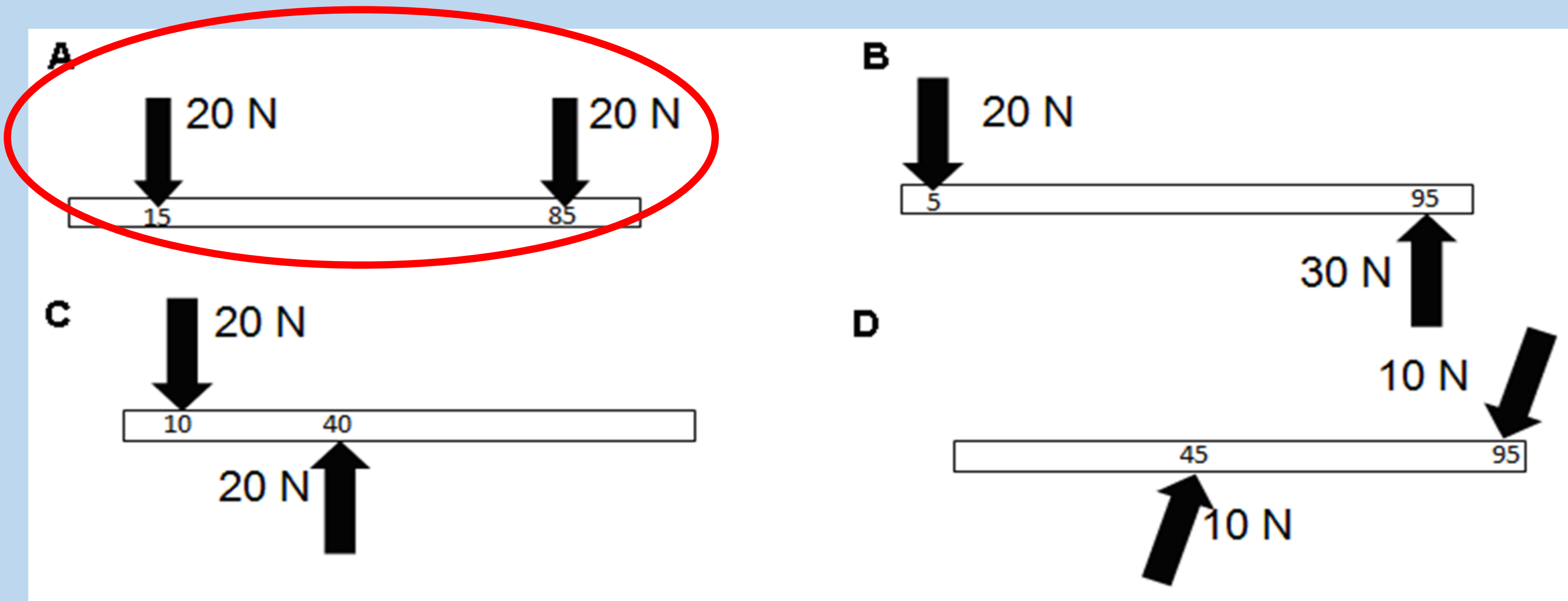
Example 11

The following diagrams show forces acting on uniform metre rules. The rulers are lying on a horizontal smooth table top and shown as seen from above.

Which of the diagrams do not show a couple?

The forces in a couple must be opposite in direction.

Diagram A does not show a couple



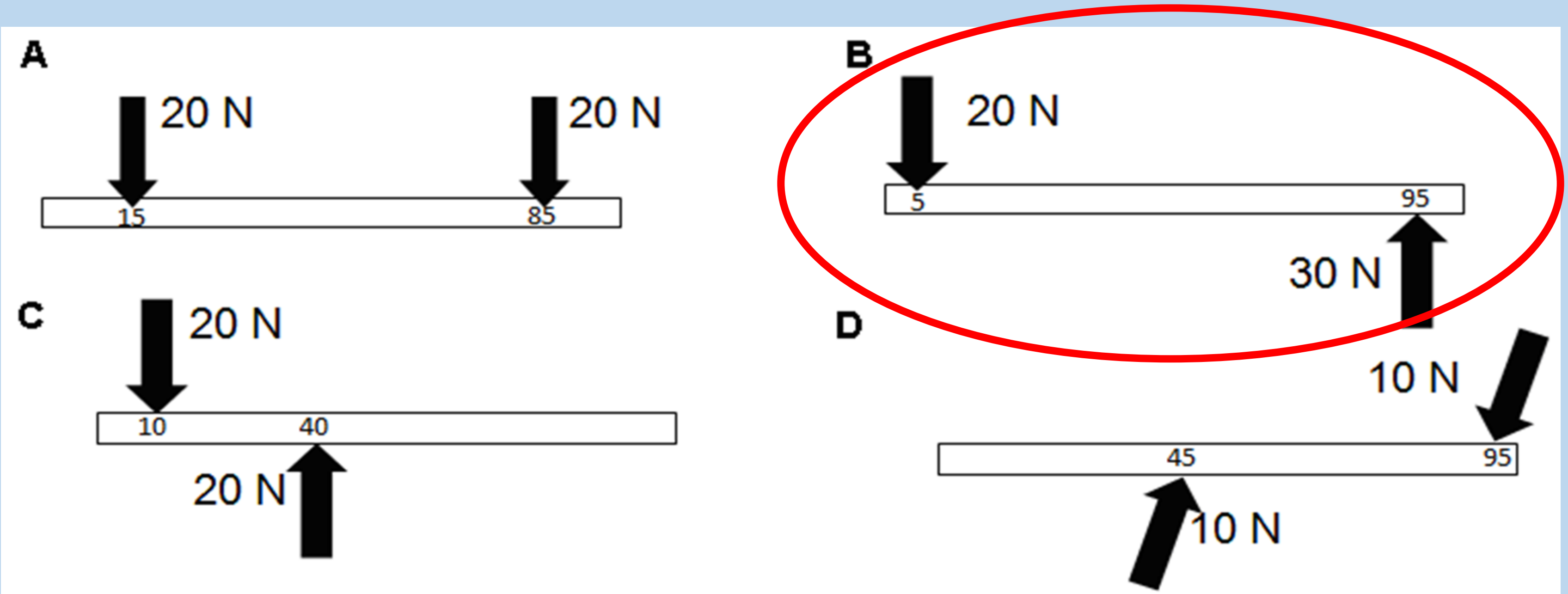
Example 11

The following diagrams show forces acting on uniform metre rules. The rulers are lying on a horizontal smooth table top and shown as seen from above.

Which of the diagrams do not show a couple?

The forces in a couple must be equal in magnitude.

Diagram B does not show a couple



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Which of the diagrams do not show a couple?

For the other diagrams, what is the torque of the couple?

For diagram C,

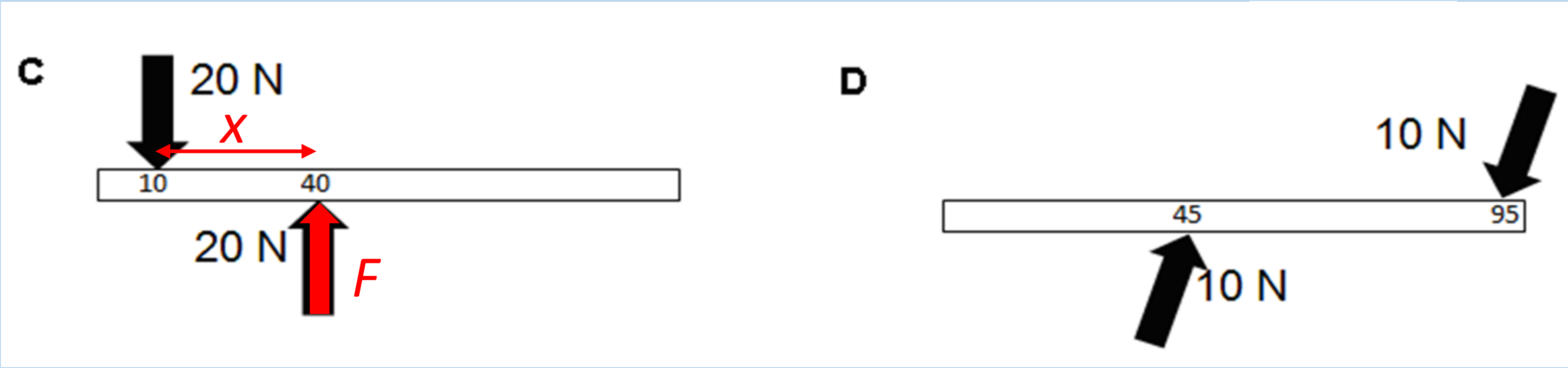
the torque due to the couple = $x\,F$

= (0.30 m) (20 N)

= 6.0 N m

40 – 10 = 30 cm

Convert to metres to get x = 0.30 m



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Which of the diagrams do not show a couple?

For the other diagrams, what is the torque of the couple?

For diagram D,

the torque due to the couple = $x\,F$

$= (0.50 \cos\theta)\,(10)$

$= (5.0 \cos\theta)$ N m

95 – 45 = 50 cm = 0.50 m

x is the *perpendicular* distance between the forces