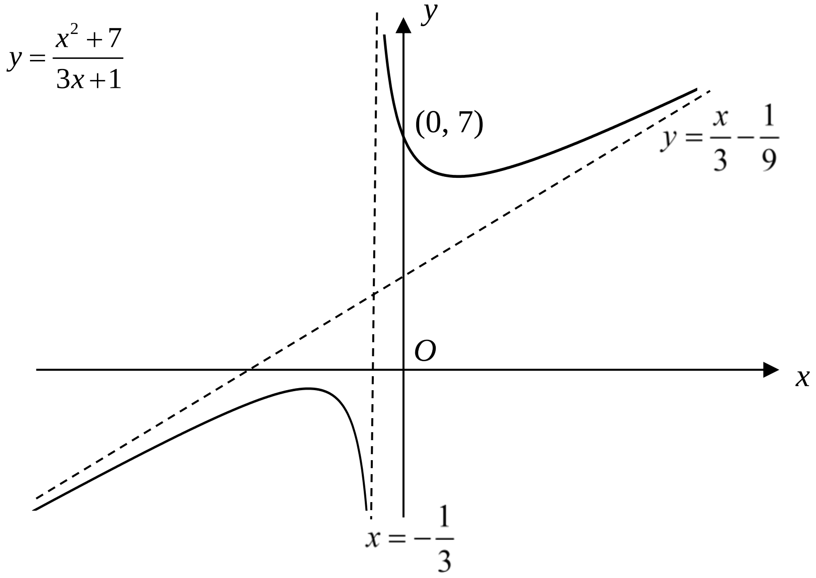
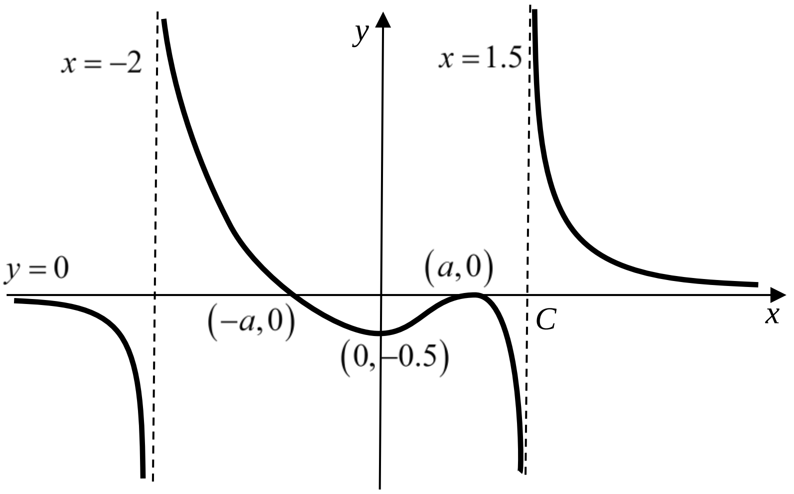
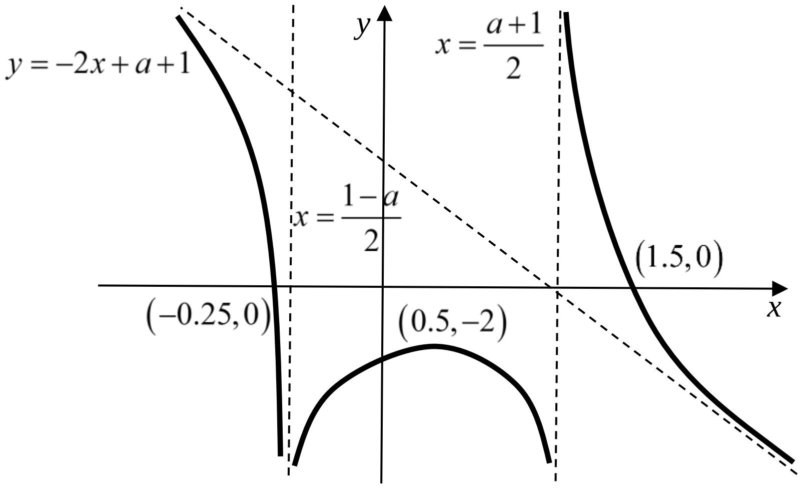
2022 ACJC H2 Math Promo Marking Scheme

Qn	Solution	Remarks
1(i)	$y=\ln\left(\frac{\mathrm{e}^{\sqrt{x}}}{\cos^3 x}\right)$ $=\sqrt{x}\ln\mathrm{e}-\ln\left(\cos^3 x\right)$ $=\sqrt{x}-3\ln\left(\cos x\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}-3\frac{-\sin x}{\cos x}$ $=\frac{1}{2\sqrt{x}}+3\tan x$	**Setter: YXF** **M1** apply laws of logarithm **A1**
(ii)	$y^{\frac{1}{x}}=x^{\ln x}$ $\frac{1}{x}\ln y=\ln x\ln x$ $\ln y=x\left(\ln x\right)^2$ $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\ln x\right)^2+2x\ln x\left(\frac{1}{x}\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=y\ln x(\ln x+2)$ **Alternative:** $y^{\frac{1}{x}}=x^{\ln x}$ $\frac{1}{x}\ln y=\ln x\ln x$ $\frac{1}{x}\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{1}{x^2}\ln y=2\frac{1}{x}\ln x$ $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{1}{x}\ln y=2\ln x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=y\left(2\ln x+\frac{1}{x}\ln y\right)$	**B1** apply ln on both sides and apply laws of logarithm **M1** apply implicit differentiation **A1** **OR** **B1** apply ln on both sides and apply laws of logarithm **M1** apply implicit differentiation **A1**

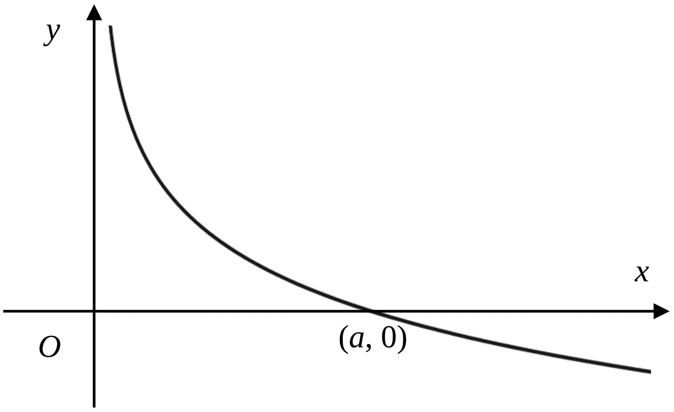
2(i)	$51-\frac{88}{x+2}\le 10x, \qquad x\ne -2$ $\frac{51(x+2)-88-10x(x+2)}{x+2}\le 0$ $\frac{51x+14-10x^2-20x}{x+2}\le 0$ $\frac{-10x^2+31x+14}{x+2}\le 0$ $\frac{10x^2-31x-14}{x+2}\ge 0$ $\frac{(2x-7)(5x+2)}{x+2}\ge 0$ $-2<x\le -\frac{2}{5}$ or $x\ge \frac{7}{2}$	**Setter: NSH** **M1** move to LHS and combine **M1** factorising the numerator **A1**
(ii)	$51-\frac{88\lvert x\rvert}{1+2\lvert x\rvert}\le \frac{10}{\lvert x\rvert}$ or $x=0$ $51-\frac{88}{\frac{1}{\lvert x\rvert}+2}\le \frac{10}{\lvert x\rvert}$ or $x=0$ $-2<\frac{1}{\lvert x\rvert}\le -\frac{2}{5}$ or $\frac{1}{\lvert x\rvert}\ge \frac{7}{2}$ or $x=0$ No solution. $-\frac{2}{7}\le x\le \frac{2}{7}$ $\therefore -\frac{2}{7}\le x\le \frac{2}{7}$	**M1** replace x with $\frac{1}{\lvert x\rvert}$ **A1**
3(i)	$y=\frac{ax^2+bx+c}{3x+1}$ Substitute $(-1,-4)$ and $(-3,-2)$ into equation, $a-b+c=8$ ----- (1) $9a-3b+c=16$ ----- (2) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(2ax+b)(3x+1)-3(ax^2+bx+c)}{(3x+1)^2}$ Substitute $x=-3$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, $0=(-6a+b)(-8)-3(9a-3b+c)$ $21a+b-3c=0$ ----- (3) From GC, $a=1,$ $b=0$ and $c=7$.	**Setter: NSH** **M1** substituting in $(-1,-4)$ or $(-3,-2)$. **M1** $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ **A1**

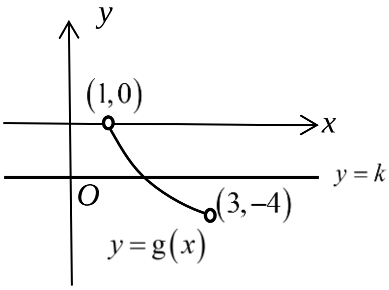
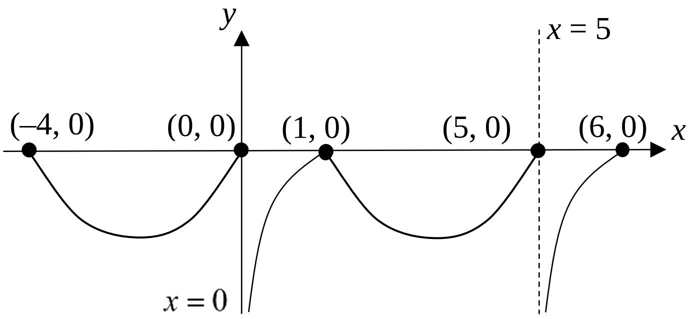
(ii)	$y=\frac{x^2+7}{3x+1}$ $=\frac{x}{3}-\frac{1}{9}+\frac{64}{9(3x+1)}$ $\begin{array}{r} \frac{x}{3}-\frac{1}{9} \\ 3x+1\overline{)x^2+0x+7} \\ x^2+\frac{x}{3} \\ \hline -\frac{x}{3}+7 \\ -\frac{x}{3}-\frac{1}{9} \\ \hline 7\frac{1}{9} \end{array}$ 	**B1** shape **B1** vertical asymptote and *y*-intercept [ECF allowed based on value of *c* found in part (i).] **B1** equation of oblique asymptote
4(i)		**Setter: YKX** **B1** equations of asymptotes **B1** shape of graph **B1** coordinates of axes intercepts (Do not penalise if not in coordinate form.)

(ii)		**B1** equations of asymptotes **B1** shape of graph **B1** coordinates of axes intercept and turning point (Do not penalise if not in coordinate form.)
5(i)	$\overrightarrow{ON}=\begin{pmatrix}6-2\lambda\\ \lambda\\ 1-\lambda\end{pmatrix}$ $\overrightarrow{OA}=\begin{pmatrix}3\\6\\1\end{pmatrix}$ $\therefore\ \overrightarrow{AN}=\begin{pmatrix}3-2\lambda\\ -6+\lambda\\ -\lambda\end{pmatrix}$ $\overrightarrow{AN}\bullet\begin{pmatrix}-2\\1\\-1\end{pmatrix}=0$ $\begin{pmatrix}3-2\lambda\\ -6+\lambda\\ -\lambda\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\-1\end{pmatrix}=0$ $-6+4\lambda-6+\lambda+\lambda=0$ $\lambda=2$ $\overrightarrow{ON}=\begin{pmatrix}2\\2\\-1\end{pmatrix}$	**Setter: LCH** **B1:** $\overrightarrow{AN}$ **M1:** $\overrightarrow{AN}\cdot\begin{pmatrix}-2\\1\\-1\end{pmatrix}=0$ **A1**
(ii)	$\overrightarrow{OA'}=2\overrightarrow{ON}-\overrightarrow{OA}$ $=2\begin{pmatrix}2\\2\\-1\end{pmatrix}-\begin{pmatrix}3\\6\\1\end{pmatrix}=\begin{pmatrix}1\\-2\\-3\end{pmatrix}$	**M1** ratio theorem or addition of vectors **A1**

(iii)	Let *D* denote the point(s) that are $3\sqrt{3}$ units away from *A* on line *L*. Then $\overrightarrow{OD}=\begin{pmatrix}6-2\lambda\\ \lambda\\ 1-\lambda\end{pmatrix}$ and $\overrightarrow{AD}=\begin{pmatrix}3-2\lambda\\ -6+\lambda\\ -\lambda\end{pmatrix}$. (can be taken from (i)). $\left\lvert\begin{pmatrix}3-2\lambda\\ -6+\lambda\\ -\lambda\end{pmatrix}\right\rvert=3\sqrt{3}$ $(3-2\lambda)^2+(-6+\lambda)^2+(-\lambda)^2=27$ $6\lambda^2-24\lambda+18=0$ $\lambda^2-4\lambda+3=0$ $(\lambda-3)(\lambda-1)=0$ $\lambda=1 \quad \text{or} \quad \lambda=3$ $\therefore D(4,1,0) \quad \text{or} \quad D(0,3,-2)$	**M1** distance formula to find λ **A1** both coordinates
6(i)	$\mathrm{f}(2(r-1))-\mathrm{f}(2r)=\cos(2r-2)\theta-\cos 2r\theta$ $=-2\sin(2r-1)\theta\sin(-\theta)$ $=2\sin(2r-1)\theta\sin\theta$ $k=2$	**Setter: YKX** **B1** usage of factor formula **B1** $k=2$
(ii)	$\sum_{r=1}^{n}\sin\left[(2r-1)\theta\right]=\frac{1}{2\sin\theta}\sum_{r=1}^{n}\mathrm{f}(2(r-1))-\mathrm{f}(2r)$ $=\frac{1}{2\sin\theta}\left[\begin{array}{ll}\mathrm{f}(0) & \cancel{-\mathrm{f}(2)}\\ \cancel{\mathrm{f}(2)} & \cancel{-\mathrm{f}(4)}\\ \cancel{\mathrm{f}(4)} & \cancel{-\mathrm{f}(6)}\\ \ldots & \ldots\\ \cancel{\mathrm{f}(2(n-3))} & -\cancel{\mathrm{f}(2(n-2))}\\ \cancel{\mathrm{f}(2(n-2))} & -\cancel{\mathrm{f}(2(n-1))}\\ \cancel{\mathrm{f}(2(n-1))} & -\mathrm{f}(2n)\end{array}\right]$ $=\frac{1}{2\sin\theta}(\cos 0-\cos 2n\theta)$ $=\frac{1}{2\sin\theta}(1-\cos 2n\theta)$ $=\frac{1}{2\sin\theta}\left(2\sin^2 n\theta\right)$ $=\frac{\sin^2 n\theta}{\sin\theta}$	√**B1** using (i) **M1** method of difference **A1** cosine double angle formula leading to given answer

(iii)	$\sum_{r=1}^{n}\sin\left[(2r+1)\theta\right]=\sum_{r-1=1}^{r-1=n}\sin\left[\left(2(r-1)+1\right)\theta\right]$ $=\sum_{r=2}^{n+1}\sin\left[(2r-1)\theta\right]$ $=\frac{\sin^2\left[(n+1)\theta\right]}{\sin\theta}-\sin\theta$	**M1** changing running index **A1** final answer
7(a)	$\overrightarrow{OE}=\overrightarrow{OA}+\lambda\overrightarrow{AB}$ $=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$ $=(1-\lambda)\mathbf{a}+\lambda\mathbf{b}$ $\overrightarrow{OE}=\overrightarrow{OC}+\mu\overrightarrow{CD}$ $=\frac{2}{3}\mathbf{a}+\mu\left(\mathbf{6b}-\frac{2}{3}\mathbf{a}\right)$ $=\frac{2}{3}(1-\mu)\mathbf{a}+6\mu\mathbf{b}$ Comparing the coefficients of **a** and **b**, $(1-\lambda)=\frac{2}{3}(1-\mu)$ $\lambda=6\mu$ $\lambda=\frac{6}{16},\ \ \mu=\frac{1}{16}$ $\overrightarrow{OE}=\left(1-\frac{6}{16}\right)\mathbf{a}+\frac{6}{16}\mathbf{b}=\frac{5}{8}\mathbf{a}+\frac{\mathbf{3}}{\mathbf{8}}\mathbf{b}$	**Setter: LCH** **B1** $\overrightarrow{OE}$ (at least 1 correct) or equivalently equation of any one of the two lines. **M1** comparing coefficients **A1**
(b)(i)	Let the foot of perpendicular from point *D* to line *OE* be *N*, with position vector **n**. Then $\mathbf{n}=\frac{(\mathbf{d}\cdot\mathbf{e})}{\lvert\mathbf{e}\rvert}\frac{\mathbf{e}}{\lvert\mathbf{e}\rvert}$ $\mathbf{f}=2\frac{(\mathbf{d}\cdot\mathbf{e})}{\lvert\mathbf{e}\rvert}\frac{\mathbf{e}}{\lvert\mathbf{e}\rvert}-\mathbf{d}$ $=2\frac{\pm3}{4}\mathbf{e}-\mathbf{d}$ $=\pm\frac{3}{2}\mathbf{e}-\mathbf{d}$	**B1** obtaining foot of perpendicular using vector projection **M1** applying ratio theorem or addition of vectors **A1** both answers
(ii)	$\frac{1}{2}(OD)(OF)=\frac{1}{2}\left\lvert\mathbf{d}\times\left(\pm\frac{3}{2}\mathbf{e}-\mathbf{d}\right)\right\rvert=\frac{3}{4}\lvert\mathbf{d}\times\mathbf{e}\rvert$ **OR** $2\left(\frac{1}{2}\right)(OE)(DE)=2\left(\frac{1}{2}\right)\left\lvert\frac{\mathbf{d}\cdot\mathbf{e}}{2}\right\rvert\left\lvert\frac{\mathbf{d}\times\mathbf{e}}{2}\right\rvert=\frac{3}{4}\lvert\mathbf{d}\times\mathbf{e}\rvert$	**M1** area of triangle formula **A1** value of *k*

8(i)	$\frac{dx}{dt} = -\frac{a}{t^2}$ and $\frac{dy}{dt} = \frac{a}{t}$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{a}{t} \times \left(-\frac{t^2}{a}\right) = -t$ When $t = p$, $x = \frac{a}{p}$, $y = a\ln p$ and $\frac{dy}{dx} = -p$ Equation of tangent: $y - a\ln p = -p\left(x - \frac{a}{p}\right)$ $y = -px + a + a\ln p$ Equation of normal: $y - a\ln p = \frac{1}{p}\left(x - \frac{a}{p}\right)$ $y = \frac{1}{p}x - \frac{a}{p^2} + a\ln p$	**Setter: YXF** **M1** $\frac{dy}{dx}$ **M1** – finding equation using formula or substituting to find *c*. **A1** equation of tangent and normal
(ii)	When $x = 0,\ y = a + a\ln p$ When $x = 0,\ y = -\frac{a}{p^2} + a\ln p$ Area of *APB* $= \frac{1}{2}\left(a + a\ln p + \frac{a}{p^2} - a\ln p\right)\left(\frac{a}{p}\right)$ $= \frac{1}{2}\left(a + \frac{a}{p^2}\right)\left(\frac{a}{p}\right)$ $= \frac{a^2(p^2+1)}{2p^3}$	**M1** – subtracting the y-intercepts or finding lengths using pythagoras theorem **A1**
(iii)		**B1** correct shape with x-intercept labelled
(iv)	Equation of tangent when $p = 1$: $y = -x + a + a\ln 1$ $= -x + a$ Note that both the tangent and the line $y = mx + a$ pass through the point (0, *a*). Range of *m*: $-1 < m < 0$.	√**B1** equation of tangent **B1** correct range of *m*

9(i)	$\mathrm{f}\left(x\right)=\left(x-1\right)\left(x-5\right)=\left(x-3\right)^2-4$ $\mathrm{R_f}=\left[-4,0\right]$	**Setter: NSH** **B1**
9(ii)	The line $y=k$ where $k\in\mathbb{R}$ cuts the graph of $y=\mathrm{g}\left(x\right)$ at most once. Hence g is one-one and g^{-1} exists. 	**B1**
(iii)	$y=\left(x-3\right)^2-4$ $\left(x-3\right)^2=y+4$ $x=3\pm\sqrt{y+4}$ $=3-\sqrt{y+4}\quad\because x<3$ $\mathrm{g}^{-1}(x)=3-\sqrt{x+4},\ \mathrm{D}_{\mathrm{g}^{-1}}=(-4,0)$	**M1** making *x* the subject **A1** for $\mathrm{g}^{-1}(x)$ **A1** for $\mathrm{D}_{\mathrm{g}^{-1}}$
(iv)	$\mathrm{h}\left(\frac{11}{2}\right)+\mathrm{h}(-2)$ $=\mathrm{h}\left(\frac{1}{2}\right)+\mathrm{h}(3)$ $=\ln\left(\frac{1}{2}\right)-4$	**M1** Getting $\mathrm{h}\left(\frac{1}{2}\right)$ or $\mathrm{h}(3)$. **A1**
(v)		**B1** one cycle drawn **B1** 2nd cycle drawn **B1** end points and asymptotes labelled
10(i)	Equation of planes are: $\mathbf{r}\bullet\frac{\begin{pmatrix}-6\\-4\\2\end{pmatrix}}{\sqrt{56}}=\frac{4}{\sqrt{56}}\pm10$ $\mathbf{r}\bullet\frac{\begin{pmatrix}-3\\-2\\1\end{pmatrix}}{\sqrt{14}}=\frac{2}{\sqrt{14}}\pm10$	**Setter: LCH** **M1** dividing by magnitude of normal to both sides, or any equivalent method **A1** equations of both planes

(ii)	$\theta=\cos^{-1}\dfrac{\begin{pmatrix}-6\\-4\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\-1\\2\end{pmatrix}}{\left\lvert\begin{matrix}-6\\-4\\2\end{matrix}\right\rvert\left\lvert\begin{matrix}-1\\-1\\2\end{matrix}\right\rvert}=\cos^{-1}\dfrac{14}{\sqrt{56}\sqrt{6}}=40.2^\circ$	**M1** formula to find angle between 2 planes **A1**
(iii)	$\begin{pmatrix}-6\\-4\\2\end{pmatrix}\times\begin{pmatrix}-1\\-1\\2\end{pmatrix}=\begin{pmatrix}-6\\10\\2\end{pmatrix}=2\begin{pmatrix}-3\\5\\1\end{pmatrix}$ Let $z=0$, $-6x-4y=4------(1)$ $-x-y=k\Rightarrow x=-k-y----(2)$ Subst. (2) into (1) $-6(-k-y)-4y=4$ $2y=4-6k\Rightarrow y=2-3k$ $\therefore x=-k-(2-3k)\Rightarrow x=2k-2$ $\therefore L_1:\mathbf{r}=\begin{pmatrix}2k-2\\2-3k\\0\end{pmatrix}+\lambda\begin{pmatrix}-3\\5\\1\end{pmatrix},\ \lambda\in\mathbb{R}$	**M1** cross product of normals to find direction of line **M1** Let $z=0$ to find position vector that lies on both planes **A1** working leading to show
(iv)	$P_3:5x+\beta y+5z=\mu$ $P_3:\mathrm{r}\bullet\begin{pmatrix}5\\\beta\\5\end{pmatrix}=\mu$ $\begin{pmatrix}-3\\5\\1\end{pmatrix}\bullet\begin{pmatrix}5\\\beta\\5\end{pmatrix}=0$ $-15+5\beta+5=0$ $\beta=2$ Subst. $\mathbf{r}=\begin{pmatrix}2k-2\\2-3k\\0\end{pmatrix}$ into $P_3:\mathrm{r}\bullet\begin{pmatrix}5\\2\\5\end{pmatrix}=\mu$ $\begin{pmatrix}2k-2\\2-3k\\0\end{pmatrix}\bullet\begin{pmatrix}5\\2\\5\end{pmatrix}=\mu$ $\mu=4k-6$	**B1** **B1**
(v)	$\beta=2$, $\mu\neq 4k-6$	√**B1** for both, ecf

<table>
<tr>
<td>11(i)</td>
<td>$\$\left(7500(1.02)-x\right)$</td>
<td>Setter: YKX
B1</td>
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<tr>
<td>(ii)</td>
<td>
<table>
<tr><td>1^{st} Nov 2022
(1st repayment)</td><td>$7500(1.02)-x$</td></tr>
<tr><td>1st Dec 2022
(2nd repayment)</td><td>$\left(7500(1.02)-x\right)(1.02)-x$
$=7500(1.02)^2-x(1.02)-x$
$=7500(1.02)^2-x(1+1.02)$</td></tr>
<tr><td>1^{st} Jan 2023
(3rd repayment)</td><td>$\left(7500(1.02)^2-x(1+1.02)\right)(1.02)-x$
$=7500(1.02)^3-x\left(1+1.02+1.02^2\right)$</td></tr>
<tr><td><i>n</i>th repayment</td><td>$7500(1.02)^n-x\left(1+1.02+...+1.02^{n-1}\right)$</td></tr>
</table>
$7500(1.02)^n-x\left(1+1.02+...+1.02^{n-1}\right)$
$=7500(1.02)^n-x\left(\dfrac{1.02^n-1}{1.02-1}\right)$
$=7500(1.02)^n-50x\left(1.02^n\right)+50x$
$=(7500-50x)(1.02)^n+50x$
</td>
<td>B1 sequence of <i>n</i> terms, seen or implied

B1 applying sum of GP

B1 working leading to given answer</td>
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<td>(iii)</td>
<td>$(7500-50x)(1.02)^{60}+50x\le 0$
By GC, $x\ge 215.75974\approx \$215.76$.

<u>OR</u>

$(7500-50x)(1.02)^{60}+50x\le 0$
$\left(50-50(1.02)^{60}\right)x\le -7500(1.02)^{60}$
$x\ge \dfrac{-7500(1.02)^{60}}{50-50(1.02)^{60}}\approx \$215.76$

$215.75974(60)-7500=\$5445.58$
<u>OR</u>
$215.76(60)-7500=\$5445.60$</td>
<td>M1 forming inequality

A1 215.76

M1 60<i>x</i>-7500

A1 either answer</td>
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<td>(iv)</td>
<td>Substituting in $x=500$, $n=12$, the amount Antonio still owes is $\left(7500-50(500)\right)(1.02)^{12}+50(500)=2805.768595$.</td>
<td>M1 Substituting in values of <i>x</i> and <i>n</i> (<i>n</i> may be wrong)</td>
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$2805.77(1.01)^n - 500\left(1+1.01+...+1.01^{n-1}\right)$

$= 2805.77(1.01)^n - 500\left(\frac{1.01^n - 1}{1.01-1}\right)$

$= 2805.77(1.01)^n - 50000\left(1.01^n\right) + 50000$

$= (2805.77 - 50000)(1.01)^n + 50000$

<table>
<tr><th>n</th><th>$(2805.77-50000)(1.01)^n + 50000$</th></tr>
<tr><td>5</td><td>398.39</td></tr>
<tr><td>6</td><td>-97.63 √</td></tr>
<tr><td>7</td><td>-598.6</td></tr>
</table>

Antonio's last repayment will be on 1st April 2024, and the amount he will be repaying is $402.37.

<u>OR</u>

$(2805.77-50000)(1.01)^n + 50000 \le 0$

$(1.01)^n \ge \frac{-50000}{2805.77-50000}$

$n \ge \frac{\ln 1.059451547}{\ln 1.01} \approx 5.80$

Least n is 6.

After the 5th repayment, Antonio will still owe $(2805.77-50000)(1.01)^5 + 50000 = 398.39$, and with the 1% interest charged on this amount, he will have to repay $402.37 on 1st April 2024.

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M1 Recalculating general term for what Antonio owes after repayments (substituting in new ratio, new <i>a</i>)

M1 using GC to solve their new equation/inequality

A1 1st April 2024 and $402.37

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<td>(v)</td>
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Let the number of additional tables (on top of 10 tables) be <i>t</i>.

Gomez Hotel cost: $1500(t+10)$.

Grande Hotel cost: $20000 + \frac{t}{2}\left(2(1950) + (t-1)(-50)\right)$.

By GC,

<table>
<tr><th>t</th><th>$1500(t+10)$</th><th>$20000 + \frac{t}{2}\left(2(1950)+(t-1)(-50)\right)$</th></tr>
<tr><td>26</td><td>54000</td><td>54450</td></tr>
<tr><td>27</td><td>55500</td><td>55100 √</td></tr>
<tr><td>28</td><td>57000</td><td>55700</td></tr>
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The couple should need at least 37 tables.

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M1 Sum of AP formula (first term and number of terms may be wrong, common difference should be correct)

M1 GC table or any other method

A1 37

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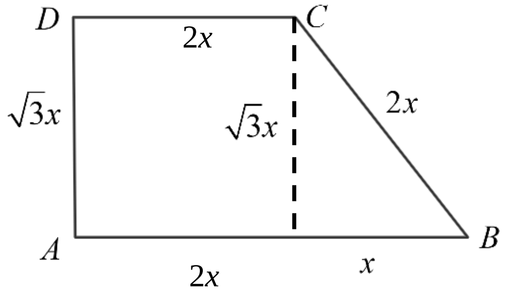
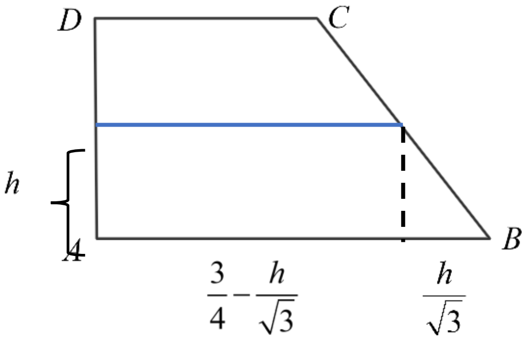
12(i)	 $AD = x\tan 60^\circ = \sqrt{3}x$ $BC = \frac{x}{\cos 60^\circ} = 2x$ $V = \frac{1}{2}(2x+3x)\times\sqrt{3}x\times y$ $5\sqrt{3}k = \frac{5\sqrt{3}x^2}{2}y$ $y = \frac{2k}{x^2}$ $A = \frac{1}{2}(2x+3x)\times\sqrt{3}x\times 2+3xy+2xy+\sqrt{3}xy$ $= 5\sqrt{3}x^2+\left(5+\sqrt{3}\right)xy$ $= 5\sqrt{3}x^2+\left(5+\sqrt{3}\right)x\left(\frac{2k}{x^2}\right)$ $= 5\sqrt{3}x^2+\frac{2\left(5+\sqrt{3}\right)k}{x}$ (Shown)	**Setter: YXF** **B1** either *AD* or *BC* **M1** expressing *V* in terms of *x* and *y* **M1** expressing *A* in terms of *x* and *y* **A1** substitution of *y* and correct working leading to answer
(ii)	For minimum *A*, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ $10\sqrt{3}x-\frac{2\left(5+\sqrt{3}\right)k}{x^2} = 0$ $5\sqrt{3}x = \frac{\left(5+\sqrt{3}\right)k}{x^2}$ $x^3 = \frac{\left(5+\sqrt{3}\right)k}{5\sqrt{3}}$ $x = \left(\left(\frac{\sqrt{3}}{3}+\frac{1}{5}\right)k\right)^{\frac{1}{3}}$ $\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = 10\sqrt{3}+\frac{4\left(5+\sqrt{3}\right)k}{x^3} = 10\sqrt{3}+\frac{4\left(5+\sqrt{3}\right)k}{\frac{\left(5+\sqrt{3}\right)k}{5\sqrt{3}}} = 30\sqrt{3} > 0$	**B1** correct $\frac{\mathrm{d}A}{\mathrm{d}x}$ **M1** $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ and making *x* the subject **A1** **B1** for checking minimum using 2nd derivative test or 1st derivative test.

	Hence, A is minimum when $x=\left(\left(\frac{\sqrt{3}}{3}+\frac{1}{5}\right)k\right)^{\frac{1}{3}}$.	
(iii)	When $k=\frac{3}{160}$ and $y=0.6$, $0.6=\frac{2}{x^2}\times\frac{3}{160}$ => $x=0.25$ Hence, $AB=0.25\times3=0.75=\frac{3}{4}$ and $AD=\sqrt{3}\times0.25=\frac{\sqrt{3}}{4}$  V now denotes the volume of water in the fish tank. $V=\frac{1}{2}\left(\frac{3}{4}-\frac{h}{\sqrt{3}}+\frac{3}{4}\right)\times h\times0.6$ $=0.3h\left(\frac{3}{2}-\frac{h}{\sqrt{3}}\right)$ $=0.45h-\frac{0.3}{\sqrt{3}}h^2$ When the fish tank is half filled, $V=\frac{1}{2}\left(5\sqrt{3}\times\frac{3}{160}\right)=\frac{3\sqrt{3}}{64}$ $0.45h-\frac{0.3}{\sqrt{3}}h^2=\frac{3\sqrt{3}}{64}$ $\frac{0.3}{\sqrt{3}}h^2-0.45h+\frac{3\sqrt{3}}{64}=0$ $h=0.19507$ *or* $h=2.40301$ (rejected since $h<AD\approx0.433$) $\frac{dV}{dh}=0.45-\frac{0.6}{\sqrt{3}}h$ $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $0.015=\left(0.45-\frac{0.6}{\sqrt{3}}\times0.19507\right)\times\frac{dh}{dt}$ $\frac{dh}{dt}=0.03922$	**B1** finding x **M1** finding V in terms of h (with value of x found earlier) **M1** finding h when the fish tank is half filled **M1** for applying chain rule to find $\frac{dh}{dt}$ **A1**

	Hence the rate of change of height is 0.03922 m per minute.	